A NEW METHOD IN FIXED POINT THEORY

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By modifying the construction of the Cartan-Leray spectral sequence of a covering [1, Chapter 16, §8], it is possible to prove the following result.

THEOREM 1. *Let π be a finite group acting on a space X. Let A be a closed π-stable subspace of X. Then there is a convergent spectral sequence with*

$$E_2^{i,j}(X, A) = \hat{H}^i(\pi, H^j(X, A))$$

where $\hat{H}^i$ denotes the Tate cohomology of π [1, Chapter 12] *and $H^j(X, A)$ is the jth Čech cohomology group of (X, A) with arbitrary coefficients and with π action induced by that on X. The term E_∞ is the graded module associated with a certain filtered module $J^*(X, A)$. This sequence is natural with respect to π-equivariant maps of (X, A). If π acts trivially on X, the sequence is trivial i.e. $E_2 = E_\infty$.*

In order to compute $J^*(X, A)$, we must impose rather drastic conditions.

THEOREM 2. *Let X be a paracompact Hausdorff space with the property that any (open) covering of X has a finite dimensional refinement. Assume that any point of X is either left fixed by all elements of π or else is such that all its images under elements of π are distinct.*

Let X^π, A^π be the sets of fixed points in X and A respectively. Then the inclusion i: $(X^\pi, A^\pi) \rightarrow (X, A)$ induces an isomorphism

$$i^*: J^*(X, A) \rightarrow J^*(X^\pi, A^\pi).$$

REMARK. i^* need not be an isomorphism of filtered modules i.e. $F^kJ^*(X, A) \rightarrow F^kJ^*(X^\pi, A^\pi)$ need not be onto for all k.

Now, assume that π is cyclic of prime order p. The condition of Theorem 2 about the action of π is then trivially satisfied. The spectral sequence is annihilated by multiplication by p, the order of π. Consequently, all terms are Z_p-modules.

THEOREM 3. *Let (X, A) and π satisfy the conditions of Theorem 2 with π cyclic of prime order p. Then for all integers k and n, and any coefficient group,*

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