An introduction to the theory of integration. By A. C. Zaanen. New York, Interscience, 1958. 9+254 pp. $7.25.

In his preface to this graduate textbook Zaanen says, "I have sailed an intermediate course between the measure approach and the linear functional approach, fully realizing the danger that the attempt to do so may not find favour in the eyes of the extreme adherents of either school." Let me say at once that the attempt does find favor in my eyes. I would predict, however, that it will be better received by the measure school than by the linear functional school. Indeed, I would classify it informally as "the Daniell integral for measure theorists." As such I think it is a welcome addition to the literature.

There naturally arises the "hen or egg" question of whether Zaanen gets the integral from a measure or vice versa. Briefly, his procedure is as follows. He starts with a measure (non-negative, countably additive set function) on a semi-ring and extends it via straight Carathéodory theory of exterior measure to a measure on a σ-ring. He then takes a linear lattice of non-negative point functions on an abstract space and defines an integral as a monotone, linear functional on this lattice which is continuous at zero with respect to monotone convergence. Next, he introduces the ordinate sets for these point functions and shows that they form a semi-ring. The integral on the lattice of point functions induces a measure on the semi-ring of ordinate sets. Extension of this measure to a σ-ring of ordinate sets generates the non-negative, measurable functions and their integrals. The only delicate problem is preservation of linearity for the extended integral. For a point function with variable sign the integral is defined as usual. When all the shouting is over we have as a theorem that the summable functions form a complete metric space in which the original linear lattice is a dense set.

By way of motivating this discussion, Zaanen turns at every opportunity to the example of Lebesgue measure and Lebesgue integral. The half-open intervals form a prime example of a semi-ring, and the step functions with their integrals defined in the obvious way furnish an example through which the linear lattice with Daniell integral is introduced.

A Stieltjes-Lebesgue integral is defined as one generated by a lattice of step functions on a measure space, and a chapter is devoted to the study of these—including the case of complex integrands. After this follows the Fubini theorem, L_p spaces, the Radon-Nikodym theorem, and differentiation of set functions. In line with Zaanen's announced intention of designing a course that can be given in a limited number of lecture hours, all this is completed on p. 157.