AN ACTION OF A FINITE GROUP ON AN n-CELL WITHOUT STATIONARY POINTS

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Communicated by Deane Montgomery, November 19, 1958

If G is a transformation group on a space X, then $x \in X$ is a stationary point if $gx = x$ for every $g \in G$. It has been an open problem, proposed by Smith [5] and by Montgomery [1, Problem 39], to determine whether every compact Lie group acting on a cell or on Euclidean space has a stationary point. Smith [4; 5] has shown the answer to be in the affirmative in case G is a toral group or a finite group of prime power order. In this note we give a simplicial action of A_5, the group of even permutations on five letters, on an n-cell without stationary points. Greever [3] has recently shown that the only finite groups of order less than 60 which could possibly act simplicially on a cell without stationary points are a certain class of groups of order 36.

We wish to thank P. E. Conner for his help and encouragement.

1. The coset space $SO(3)/I$.

Let $SO(3)$ denote the group of all proper rotations of Euclidean 3-space E^3 and let $I \subset SO(3)$ be the group of rotational symmetries of the icosahedron. As a group, I is isomorphic to A_5 (see [9, pp. 16–18]) and hence is simple.

LEMMA 1. *The coset space* $SO(3)/I$ *has the integral homology groups of the* 3-*sphere* S^3.

PROOF. Let Q denote the algebra of quaternions and $Q_1 \subset Q$ the group of quaternions of norm one. Identify Q with E^4 and Q_1 with S^3. Let $\tau\colon Q_1 \to SO(3)$ be the standard homomorphism, which is a two-to-one covering map. Set $I' = \tau^{-1}(I)$. Then τ induces a homeomorphism $Q_1/I' \approx SO(3)/I$.

The natural map $\pi\colon Q_1 \to Q_1/I'$ is a covering map and the group of covering translations is given by the action of I' on Q, by right multiplication. Since every covering translation preserves orientation it follows that Q_1/I' is an orientable 3-manifold and hence $H_3(Q_1/I') \approx H_3(SO(3)/I) \approx Z$ (here Z denotes the integers).

From covering space theory the fundamental group $\pi_1(Q_1/I')$ is isomorphic to I'. Thus $H_1(Q_1/I')$ is isomorphic to $I'/[I', I']$ where $[I', I']$ denotes the commutator subgroup of I'. Since I is simple,

[1] The first named author is the holder of a NSF Senior Postdoctoral Fellowship; the work of the second has been supported in part by contract A.F. 49(638)-104.