

SOME PROBLEMS ON 3-DIMENSIONAL MANIFOLDS

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I. GENERALITIES

1. Introduction. One of the well-known problems in Topology is the *classification problem* of closed n -dimensional manifolds.

An n -manifold (n -dimensional manifold) is a connected separable metric space each of whose points has a closed neighborhood homeomorphic to a closed n -cell. So we consider both manifolds with boundary and manifolds without boundary. A *closed* n -manifold is a compact n -manifold without boundary.

Classification means to define an infinite sequence of closed n -manifolds M_1, M_2, M_3, \dots ,¹ such that any two of these are not homeomorphic, but any closed n -manifold M is homeomorphic with one of them. We emphasize that, we do not ask to find a method to decide with which of the *model* manifolds is M homeomorphic. We only want to know whether M is included in this sequence. Of course we do not ask to find an *effective procedure*, because such may not exist.

The classification problem was solved long ago for $n=2$, i.e. for closed surfaces,² [22, §§37–39, pp. 130–142]. So, as usual in Mathematics, one tries to solve the problem for the next dimension $n=3$, in the hope that he will find a general method working for any dimension. This is the reason we restrict ourselves from now on to the case $n=3$.

The classification problem has been solved not only for closed surfaces, but also for compact nonclosed ones [22, §40, pp. 142–144; 10, pp. 151–158]. See also [10, p. 171, ll. 12–16].

We concentrate our attention on the classification problem of *closed* 3-manifolds, and for the time being we do not consider the classification problem for nonclosed 3-manifolds, because this last problem seems to be much more complicated, see No. 21.

2. Generalities. As is well known, the classification problem is solved for $n=2$ by cutting the surface along simple³ curves. So the question arises naturally: *Can we solve the classification problem for*

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¹ To define M_i means to give a model of M_i , i.e. a way of constructing M_i .

² Numbers in brackets refer to the bibliography at the end of the paper.

³ I.e. without self-intersections.