REPRESENTATION OF ABSTRACT RIESZ POTENTIALS OF THE ELLIPTIC TYPE

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Using semigroup theory we are able to obtain an abstract definition of the Riesz potentials of the elliptic type as closed linear operators, as well as a representation for them without using continuation.

Let

$$T(\xi),\ \xi = (\xi_1, \xi_2, \cdots, \xi_n),\ -\infty < \xi_k < \infty,$$

be an n-parameter group (strongly continuous) of endomorphisms over a B-space X. Let

$$T(\xi) = \prod_{k=1}^{n} T_k(\xi_k),$$

each $T_k(\xi_k)$ being a strongly continuous one-parameter group with infinitesimal generator A_k. Let

$$C = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} A_i A_j,$$

where the matrix $[a_{ij}]$ is real symmetric and positive definite. Then C is the infinitesimal generator of a one-parameter strongly continuous semigroup $S(t)$, $0<t$, and further $\sup \|S(t)\| < \infty$. Now, in some previous work the author has shown that in such a case it is possible to define $(-C)^\alpha$, Re $\alpha>0$, as closed linear operators, interpolating integral powers and having the semigroup property in α. Moreover, they have an explicit representation as Bochner integrals in terms of $S(t)$, which for $0<\text{Re}\ \alpha<1$, is

$$(1) \qquad (-C)^\alpha x = \frac{1}{\Gamma(-\alpha)} \int_0^\infty [S(t)x - x] t^{-\alpha-1} dt,$$

for $x \in D(C)$. Next, to simplify the notation, let

$$C = \sum_{i=1}^{n} A_i^2.$$

Then for every $x \in X$,

$$(2) \qquad S(t)x = \frac{1}{(2(\pi t)^{1/2})^n} \int_{E_n} T(\xi)x \exp\left[-\sum_{1}^{n} \xi_k^2/4t\right] d\xi_1 \cdots d\xi_n.$$