The proofs of (4.2) and then of (1.1) are essentially the same as those of (3.2) and Dehn's lemma. The details are left to the reader.

References

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RESEARCH PROBLEMS

12. Richard Bellman: *Ordinary differential equations.*

It is known that if

a. A is a stability matrix, i.e., all characteristic roots have negative real parts,

b. $\|g(x)\|/\|x\|\to 0$ as $\|x\|\to 0$, $(\|x\|=\sum_i |x_i|)$,

then all solutions of $dx/dt=Ax+g(x)$ approach zero as $t\to\infty$, provided that $\|x(0)\|$ is sufficiently small (Poincaré-Lyapunov theorem).

If $x(0)=a_1c$, where c is a characteristic vector of A and a_1 is a scalar, what is the precise bound for $|a_1|$ in terms of A and $g(x)$? (Received January 7, 1958.)

13. Richard Bellman: *Partial differential equations.*

It is known that if $|g(u)|/|u|\to 0$ as $u\to 0$, then the solution of $u_t=u_{xx}+g(u)$, $u(0, t)=u(1, t)=0$, $t>0$, approaches zero as $t\to\infty$, provided that $\mathrm{Max}_{0\leq x\leq 1}|u(x, 0)|$ is sufficiently small.

a. If $u(x, 0)=c_1$ what is the precise bound for $|c_1|$ in terms of $g(u)$?

b. If $u(x, 0)=c_1 \sin k\pi x$, what is the precise bound for $|c_1|$ in terms of $g(u)$?

14. Richard Bellman: *Functional equations.*

Let $f_n(u)$ be an analytic function of the function $u(x)$ and its first n derivatives $u'(x), \cdots, u^{(n)}(x)$, for $u\neq 0$, satisfying the functional equation

$$f_n(uv) = f_n(u) + f_n(v).$$

It is well-known that $f_0(u)=c_1 \log u$, and under much lighter conditions, and it is easy to show that $f_1(u)=c_1\log u+c_2u'/u$.

What is the analytic form of f_n for general n? (Received January 9, 1958.)

15. Richard Bellman: *Functional equations and differential equations.*

Consider the nth order linear differential equation

$$\frac{d^nu}{dt^n}+a_1(t)\frac{d^{n-1}u}{dt^{n-1}}+\cdots+a_n(t)u=0$$