

*Fundamental concepts of algebra.* By Claude Chevalley. New York, Academic Press, 1956. 8+241 pp. \$6.80.

Chevalley has written a text-book, and his mathematical personality permeates every paragraph. Readers of his *Algebraic functions of one variable* who agreed at the time with André Weil's dictum "algebraic austerity can go no further" may decide that a counterexample has been produced. The book is tight, unified, direct, severe; relentlessly and uncompromisingly it pursues its ends: out of the simplest basic notions of algebra to build up with perfect precision the theory of the multilinear algebras of modules and to discuss those particular multilinear algebras which have found applications in topology and differential geometry. Group and ring theory are down to the irreducible minimum, field theory is completely absent; in their place, modules and their tensor products and the algebras one constructs from them: tensor, exterior, symmetric. The unity is monolithic. Gone is the discursive rambling of previous texts. This one marches unswerving and to its own music. It is presented by Chevalley as a serious effort to "adapt modern algebra teaching to present-day requirements"; since it represents thereby the first real departure in English from the van der Waerden tradition in first-year graduate algebra texts, it should be considered in some detail.

The general approach to the subject matter is that of Bourbaki's first three algebra chapters, but there are significant differences in content and treatment (Chevalley is often more general). As for the style, Bourbaki emerges from the comparison a warm, compassionate, and somewhat elderly gentleman.

The first chapter of the book is an Overture: in these first twenty pages are set forth the major themes of the entire book. They are devoted to the monoid (*née* semigroup with identity) and her cortege—submonoid, quotient monoid and homomorphism, product and free monoid. Two basic modern techniques appear and are emphasized from the very beginning here. One is the proof that  $A$  and  $B$  are isomorphic by constructing opposing maps  $f$  and  $g$  such that  $f \circ g$  and  $g \circ f$  are the identity. The other is the universal mapping property characterization, here used to define the free monoid and soon to be ubiquitous. This emphasis on mappings is one of the most characteristic features of the new algebra; the older "identifications" are now explicit natural mappings, and what earlier required brow-wrinkling now needs only diagram-chasing. So Chevalley appropriately puts in here two little diagrams and three little paragraphs explaining them. In view of the importance of diagrams to this sort of algebra, probably a few more should have been included later on as a sugges-