THE NOVEMBER MEETING IN PASADENA

The five hundred twenty-eighth meeting of the American Mathematical Society was held at the California Institute of Technology in Pasadena, California, on Saturday, November 17, 1956. Attendance was about 115, including 86 members of the Society.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, Professor Bertram Yood delivered an address on *Semi-simple Banach algebras*. He was introduced by Professor Edwin Hewitt, and the sessions for contributed papers presided over by Professors Richard Arens and R. P. Dilworth.

Following are abstracts of papers presented at the meeting, those whose numbers are followed by "*t*" having been given by title. On joint papers, the presenter's name is followed by "(p)". Mr. Bryant was introduced by Professor Roy Dubisch and Dr. Montague by Professor Alfred Tarski.

Algebra and Theory of Numbers

70*t*. B. W. Brewer: *On certain character sums and related congruences.*

The number of solutions (u, w) of the congruence $u^k+Q\equiv w^2 \bmod p$, p a prime, is given by $p+(Q|p)+\psi_k(Q)$, where $\psi_k(Q)=\sum_{u=1}^{p-1}(u^k+Q|p)$, and has been expressed in terms of certain quadratic partitions of p for $k=3$, 4, 5, 6, and 8. For $p=a^2+b^2$ $(a\equiv 1 \bmod 4)$, $\psi_4(Q)=-2a(Q^{1/2}|p)-2$ if $(Q|p)=1$, and $\psi_4(Q)=\pm 2b$ if $(Q|p)=-1$ (Jacobsthal). The ambiguity in sign in the latter case has been removed by E. Lehmer (Pacific Journal of Mathematics vol. 5 (1955) pp. 103–118) if 2 is a quartic nonresidue of p. Theorem 1: If 3 is a quadratic nonresidue of the prime $p=a^2+b^2$ $(a\equiv 1 \bmod 4$, $b\equiv a \bmod 3)$, and $(Q|p)=-1$, then $\psi_4(Q)=3b(\alpha|p)$, where $3\alpha^2+Q\equiv 0 \bmod p$. Theorem 2: If the prime $p=12k+1=a^2+b^2=s^2+3t^2$ $(a\equiv 1 \bmod 4$, $s\equiv 1 \bmod 3)$, then (1) $\psi_{12}(Q)=-2a[1+2([-3]^{1/2}|p)](\alpha|p)-4s-2$ if $Q\equiv\alpha^6 \bmod p$, (2) $\psi_{12}(Q)=-2a[1-([-3]^{1/2}|p)](\alpha|p)+2s-2$ if $Q\equiv\alpha^2\not\equiv\alpha_1^3 \bmod p$, (3) $\psi_{12}(Q)=\pm 2b[1-2([-3]^{1/2}|p)]$ if $Q\equiv\alpha^3\not\equiv\alpha^2 \bmod p$, (4) $\psi_{12}(Q)=\pm 2b[1+([-3]^{1/2}|p)]\pm 6t$ if $Q\not\equiv\alpha^2$, $Q\not\equiv\alpha_1^3 \bmod p$. Theorem 3: If the prime $p=c^2+2d^2$ (hence $p=8n+1$ or $8n+3$), then $\sum_{x=0}^{p-1}((x+2)(x^2-2)|p)=2c$, where $c\equiv(-1)^{n+1} \bmod 4$; and if $p\neq c^2+2d^2$ (hence $p=8n-1$ or $8n-3$), then $\sum_{x=0}^{p-1}((x+2)(x^2-2)|p)=0$. Theorem 3 is an exact analog of the known results for primes of the form a^2+b^2 and s^2+3t^2. (Received September 24, 1956.)

71. S. J. Bryant: *Isomorphism order for abelian groups.*

A group G is said to have isomorphism order k if G has the following property: If H is a group such that every subgroup of H which can be generated by k or fewer elements is isomorphic to a subgroup of G then H is isomorphic to a subgroup of G.

Theorem: An abelian group G has isomorphism order k if and only if G is a direct sum of two groups, one torsion the other torsion free. The torsion free summand is