BOOK REVIEWS

Homological algebra. By Henri Cartan and Samuel Eilenberg. Princeton, The Princeton University Press, 1956. 15+390 pp. $7.50.

At last this vigorous and influential book is at hand. It took nearly three years from completed manuscript to bound book; Princeton is penalized 15 yards for holding.

Homological algebra deals both with the homology of algebraic systems and with the algebraic aspects of homology theory. The first topic includes the homology and cohomology theories of groups, of associative algebras, and of Lie algebras. The second topic includes the care and feeding of exact sequences and spectral sequences, as well as the manipulation of functors of chain complexes. For example, the Künneth problem reads: Given the homology of complexes K and L, what is the homology of $K \otimes L$? Again, the universal coefficient problem reads: Given a group G and the homology of a complex K, what is the homology of the complexes $K \otimes G$ and Hom (K, G)? These problems and these two functors, tensor product and Hom, are treated not just for groups, but in proper generality for left modules over an arbitrary ring Λ. Explicitly, if A and G are such modules, $\mathrm{Hom}_\Lambda (A, G)$ denotes the group of Λ-module homomorphisms of A into B. When G is a right Λ-module and A a left module—a situation denoted neatly as $(G_\Lambda, {}_\Lambda A)$—the tensor product taken over Λ is written as $G \otimes_\Lambda A$. A Λ-complex K is a graded differential left Λ-module; its homology $H(K)$ is the usual graded module $H(K) = \sum H_n(K)$, it has the usual definition and an unusual definition (Chap. IV), dual to the usual one.

The various aspects of homological algebra all meet in the notion of a projective resolution (Chap. V). A left module P is *projective* (Chap. I) if any homomorphism of P into a quotient module B/C can be "lifted" into a homomorphism of P into B. (Thus a free module is projective, but not necessarily vice versa.) A projective resolution of A is an exact sequence $\cdots \to X_n \to X_{n-1} \to \cdots \to X_0 \to A \to 0$ composed of A and a complex X which consists of projective modules X_n, $n = 0, 1, \cdots$. Given two such resolutions X and X' for the same A, the familiar method of climbing up one dimension at a time provides a chain transformation of X into X' and proves X and X' chain equivalent. Given any module G_Λ the homology groups $H(G \otimes_\Lambda X)$ are therefore independent of the choice of the resolution of A and depend only on G and A; they are called the torsion products and are denoted (Chap. VI)