# RIEMANN'S METHOD IN THE THEORY OF SPECIAL FUNCTIONS

J. SCHWARTZ[1]

*To E. L. Post*

The theory of special functions is known as a "painful" subject. My talk today might as well be entitled, "special functions without pain." The painful nature of the subject springs from this: that at first sight, it seems to consist of an enormous number of uncorrelated individual cases, the particular significance, connection, or depth of any specific formula being hard to judge—an enormous mass of chaotic detail, the worst sort of material with which to deal. What is needed, then, is some way of ordering and correlating the material, which should, in order of importance, accomplish the following ends.

(1) To make evident in a general intuitive way what sorts of formulae are likely to hold "on standard grounds" and, conversely, which formulae depend on really special or on deeper properties of a given special function.

(2) To give in a mechanical way the exact form of as wide a class of formulae as possible.

(3) To furnish exact proofs of these formulae.

In attempting to develop such a scheme, it is clear that the question of notation will play a role of importance. In contrast to the conventionally meaningless ad hoc notations for the various kinds of special functions, one wants to develop a "natural" notation, which will bring out as much of the structure of a given function as possible. What I would like to do today is to indicate how one such system can be developed, to indicate the derivation of a few formulae on the basis of this system, and to describe in a general way the range of applicability of the methods involved.

The complete system would consist of three parts:

(a) the function theoretic method (Riemann's method);

(b) the Laplace transform method;

(c) the method of partial differential equations.

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