TOPOLOGICAL ANALYSIS

G. T. WHYBURN

1. Introduction. Apart from the really spectacular development of the field of topology during the last 50 years, one of the most interesting and satisfying related phenomena has been the widespread interaction between topology and other branches of mathematics. This has occurred in nearly all fields of mathematics. It has been notable in algebra, algebraic geometry, differential geometry and in various types of analysis. However it is the connection with the theory of functions, and particularly functions of a complex variable, which has developed and is being actively developed at present that I should like to discuss in some detail today. As I have used the term, *topological analysis* refers to those results of the analysis type, theorems about functions or mappings from one space onto another or about real or complex valued functions in particular, which are topological or pseudo-topological in character and which are obtainable largely by topological methods. Thus in a word we have analysis theorems and topological proofs. As just indicated however, what I shall say today will be confined largely to results closely related to analytic functions of a complex variable. Since one of the main roots—if indeed not the taproot— of topology rests firmly in the recognition by Riemann and Poincaré of the fundamental and inescapable topological nature of such functions, much of the work to be described represents a return of topology to some of the original situations and problems which motivated its beginnings and to which it owes much for its early development.

Contributions of fundamental concepts and results in this type of work have been made during the past 25 years by a large number of mathematicians. Among these should be mentioned (1) Stoïlow [1], the originator of the interior or open mapping, who early recognized lightness and openness as the two fundamental topological properties of the class of all nonconstant analytic functions; (2) Eilenberg [2] and Kuratowski [3] who introduced and used an exponential representation for a mapping and related it to properties of sets in a plane, (3) Morse and Heins [4], whose studies on invariance of topological indices of a function under admissible deformations of curves in the

Presidential address delivered before the Annual Meeting of the Society in Houston, Texas, December 28, 1955; received by the editors January 28, 1956.