DUALITY AND S-THEORY

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This is an account of a systematization of certain parts of homotopy theory by means of the suspension category (also called the S-category). One of the most important applications is that a formal analogy becomes a rigorous duality in the S-category. We begin by summarizing some of the background material and results leading to the S-theory and duality.

1. Given topological spaces X and Y and two continuous mappings f_0 and f_1 from X to Y (denoted by $f_0, f_1: X \to Y$) we say that f_0 is *homotopic* to f_1 (denoted by $f_0 \simeq f_1$) if there exists a continuous family of continuous mappings $h_t: X \to Y$ for $0 \leqq t \leqq 1$ such that $h_0 = f_0$ and $h_1 = f_1$. Intuitively $f_0 \simeq f_1$ if the map f_0 can be continuously deformed into the map f_1. It is easily seen [**4**; **7**] that the relation of homotopy thus defined is reflexive, symmetric, and transitive and, therefore, partitions the set of continuous maps from X to Y into disjoint equivalence classes called *homotopy classes*. We denote the set of homotopy classes of mappings from X to Y by $[X, Y]$, and if $f: X \to Y$, then $[f]$ will denote the homotopy class of f. It is of fundamental importance in present day topology to determine the structure of $[X, Y]$. Specifically, we would like to have a method of determining whether two given mappings are homotopic, and also we would like to determine how many elements there are in $[X, Y]$. In many cases our information is so limited that we do not even know whether $[X, Y]$ contains more than one element. (In the above when X is contractible we assume Y is arcwise connected.)

It is clear that if either X or Y is a contractible space then there is exactly one homotopy class of maps from X to Y. In particular if either X or Y is a cell of any dimension, the structure of $[X, Y]$ is completely known as this set consists of a single element.

Perhaps the most natural spaces to consider after the cells are the spheres. We denote by S^n the unit sphere in a euclidean space of $(n+1)$ dimensions. We want to discuss the homotopy classes $S^n \to Y$ and $X \to S^n$.

An address delivered before the Milwaukee meeting of the Society, November 25, 1955 under the title *Aspects of duality in homotopy theory*, by invitation of the Committee to Select Hour Speakers for Western Sectional Meetings; received by the editors January 4, 1956.