and the last chapter treats the difficult topic of periodic solutions on a torus.

An important feature of the book is the inclusion of approximately one hundred and seventy-seven problems of varying degrees of difficulty, with hints as to the solution. This greatly increases the scope of the book. Finally, let us note that the book is printed in the attractive easy-on-the-eyes style which we have grown to expect from McGraw-Hill.

RICHARD BELLMAN

Elemente der Funktionalanalysis. By L. A. Ljusternik and W. I. Sobolew. (Mathematische Lehrbücher und Monographien, vol. 8.) Berlin, Akademie-Verlag, 1955. 10+253 pp. 25.00 DM.

This book is a translation of *Elementy funkcional'nogo analiza* [Gostehizdat, Moscow-Leningrad, 1951], which was reviewed in Mathematical Reviews in Vol. 14 (1953) p. 54. It is an introduction to the theory of normed linear spaces and operations on them, and contains few surprises. It is more complete and more sophisticated than *Elementy teorii funkciĭ i funkcional'nogo analiza* by Kolmogorov and Fomin [Izdatel'stvo Mosk. Universiteta, 1954], and is totally different from *Leçons d'analyse fonctionnelle* by Riesz and Sz.-Nagy [Akadémiai Kiadó, Budapest, 1952]. There are many points of contact with Banach's classical monograph *Théorie des opérations linéaires* [Monografje Matematyczne, Warszawa, 1932]. So far as the reviewer knows, there is no single treatise in English covering the same material as the book under review.

The reader is tacitly expected to know the elementary theory of functions of a real variable: continuity; differentiation; functions of finite variation; Lebesgue integration on $[0, 1]$. For students with this background, the book is highly recommended as an introduction to functional analysis.

Chapter I deals with metric spaces. The only nonstandard topic here is Banach's theorem on contracting mappings: a mapping A of a space X (with metric ρ) into itself such that $\rho(Ax, Ay) \leqq \alpha\rho(x, y)$ for all $x, y \in X$ and some α, $0<\alpha<1$, admits exactly one fixed point. Several applications of this theorem are given. Chapters II and III deal with linear spaces, linear operators, and linear functionals; this part bears a strong family resemblance to Banach's book. Several interesting and not universally known facts are given: for example, if E is a Banach space with conjugate space $\overline{E}$, and if E is not reflexive, then

$$E, \overline{E}, \overline{\overline{E}}, \overline{\overline{\overline{E}}}, \ldots$$