RESEARCH PROBLEMS

1. Richard Bellman: *Dynamic programming.*

Solve

$$f(x, y) = \operatorname{Max} \begin{bmatrix} p_1[r_1x + f((1 - r_1)x, y)], \\ q_1[s_1y + f(x, (1 - s_1)y)], \\ p_2[r_2x + s_2y + f((1 - r_2)x, (1 - s_2)y)] \end{bmatrix}, \qquad x, y \geqq 0,$$

where $0 < p_1, p_2, q_1, r_1, r_2, s_1, s_2 < 1$; see Proc. Nat. Acad. Sci. U.S.A. vol. 39 (1953) pp. 1077–1082; vol. 38 (1952) pp. 716–719. (Received September 23, 1954.)

2. Richard Bellman: *Probability theory.*

Let $\{Z_k\}$ be a sequence of random matrices having a common probability distribution, say p, of being A and $(1-p)$ of being B, where A and B are two matrices having all positive elements. Let $X_N = \prod_{k=1}^{N} Z_k$, and $x_{ij}(N)$ be the ijth element in X_N. Determine the limiting distribution of $\log x_{ij}(N)$, suitably normalized. See Proc. Nat. Acad. Sci. U.S.A. vol. 39 (1953) and Rand Paper 398, to appear in a forthcoming issue of Duke Math. J. (Received September 23, 1954.)

3. Richard Bellman: *Analysis.*

Let $\prod_{k,l=0}^{\infty} (1 - x^k y^l t)^{-1} = \sum_{n=0}^{\infty} q_n(x, y)t^n$. Obtain a formula for $q_n(x, y)$. (Received September 23, 1954.)

4. Richard Bellman: *Number theory.*

Let $f(x)$ be an irreducible polynomial with integer coefficients and the property that $f(x) > x$ for $x \geqq a$. Prove that the sequence $\{x_n\}$ defined by the recurrence relation $x_{n+1} = f(x_n)$, $x_0 = a$, an integer, cannot represent primes for all large n. See Bull. Amer. Math. Soc. vol. 53 (1947) pp. 778–779. (Received October 2, 1954.)

5. Richard Bellman: *Number theory.*

Let x be a rational number greater than one, and let $[y]$ denote, as customary, the greatest integer contained in y. Prove that $[x^n]$ cannot be prime for all large n. (Received October 2, 1954.)

6. Richard Bellman: *Number theory.*

Prove that $\sum_{n=1}^{N} d(n^3+2) \sim cN \log N$ as $N \to \infty$. See Duke Math. J. vol. 17 (1950) pp. 159–168. (Received October 2, 1954.)

7. A. D. Wallace: *Manifolds with multiplication.*

Let M be a compact connected manifold without boundary and provided with a continuous associative multiplication such that $MM = M$. Does the following hold: either (i) M is a group or (ii) $xy = y$ for each x, y or $xy = x$ for each x, y? It is known that (i) holds if we replace "$MM = M$" by "there is a two-sided unit"; see Summa Brasil. Math. vol. 3 (1953) pp. 43–55. (Received October 4, 1954.)

8. A. D. Wallace: *Fixed points for topological lattices.*

A topological lattice is a Hausdorff space X together with two maps (continuous