SOLVING LINEAR ALGEBRAIC EQUATIONS CAN BE INTERESTING

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1. **Introduction.** The subject of this talk is mathematically a lowly one. Consider a system of n linear algebraic equations in n unknowns, written

$$Ax = b. \tag{1}$$

Here A is a square matrix of order n, whose elements are given real numbers a_{ij} with a determinant $d(A) \neq 0$; x and b denote column vectors, and the components of b are given real numbers. (Complex numbers would offer no essential difficulty.) It is desired to calculate the components of the unique solution $x = A^{-1}b$; here A^{-1} is the inverse of A.

Such problems arise in the most diverse branches of science and technology, either directly (e.g., the normal equations of the least-squares adjustment of observations) or in an approximation to another problem (e.g., the difference-equation approximation to a self-adjoint boundary-value problem for a partial differential equation). These two are very frequent sources of numerical systems; note that $A > 0$ (i.e., A is symmetric and positive definite) in both examples. The order n is considered to range from perhaps 6 or 8 up to as large a number as can be handled. Stearn **[111]**, for instance, mentions the solution of a system of order 2300 by the U.S. Coast and Geodetic Survey. The accuracy demanded of an approximate solution ξ varies widely; even the function which is to measure the accuracy of ξ varies or is unknown. Some "customers" want to make the length $|b - A\xi|$ small; some, $|\xi - A^{-1}b|$; others have apparently thought only in terms of getting $A^{-1}b$ exactly.

We all know that each component of the solution $A^{-1}b$ can be expressed as a quotient of determinants by Cramer's rule. We have all evaluated determinants of orders 3, 4, and possibly 5, with a_{ij} integers; it is quite easy and rather boring. I therefore suspect that the average mathematician damns the practical solution of (1) as being both trivial and dull.

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