Introduction to Hilbert space and the theory of spectral multiplicity. By P. R. Halmos. New York, Chelsea, 1951. 114 pp. $3.25.

There is little doubt that the author of this book enjoyed himself thoroughly during its preparation. Reading the result afforded this reviewer considerable pleasure. In one hundred and nine well-packed pages one finds an exposition which is always fresh, proofs which are sophisticated, and a choice of subject matter which is certainly timely. Some of the vineyard workers will say that P. R. Halmos has become addicted to the delights of writing expository tracts. Judging from recent results one can only wish him continued indulgence in this attractive vice.

The present work may confidently be recommended. However, beginners in the field should be cautioned before they rush off to secure a copy. Unless one is equipped and in training, one should not attempt the expedition. One must not be misled by the title. For this introduction to Hilbert space, one has to be an expert in measure theory. As a matter of fact it is best to have read the author's book on measure theory or its equivalent. One has to know enough about Banach spaces to be conversant with the Riesz representation theorem for the linear functionals on the space of continuous functions. We would be ready to wager that most young mathematicians learn that theorem subsequent to the theorem on the spectral resolution of hermitian operators and not prior to it—which is the scheme of things here. But, for those who know this material and wish an excellent introduction to multiplicity theory, this tract is just right.

The subject matter of the book is funnelled into three chapters: The geometry of Hilbert space; the structure of self-adjoint and normal operators; and multiplicity theory for a normal operator. For the last, an expert knowledge of measure theory is indispensable. Indeed, multiplicity theory is a magnificent measure-theoretic tour de force. The subject matter of the first two chapters might be said to constitute an introduction to Hilbert space, and for these, an a priori knowledge of classic measure theory is not essential. Thus one may question the author's decision to unveil his virtuosity in this direction sooner than was necessary, or perhaps desirable.

Chapter I has some features which differentiate it from previous texts in this domain. The Hilbert spaces under consideration are not assumed separable. The handling in proofs and notation from this point of view is completely successful. For another thing, considerable use is made of the parallelogram identity for vectors: $\|f+g\|^2+\|f-g\|^2=2\|f\|^2+2\|g\|^2$. The spirit of this and similar identities has certainly pervaded many smoke-filled colloquium rooms in