

*Problèmes concrets d'analyse fonctionnelle.* By P. Lévy. Paris, Gauthier-Villars, 1951. 14+484 pp. 4000 fr.

This work is a revised edition of Lévy's *Leçons d'analyse fonctionnelle* published in 1922. The purpose of the new title is to emphasize the difference in method and point of view between this book and the very abstract and general theories to which the name "functional analysis" is now commonly attached. Apart from the fact that Lévy restricts attention to certain special function spaces, there is a more striking difference between this work and most other publications on functional analysis, in that Lévy devotes most of his space to the study of functional differential equations. These are the analogues in function space of total differential equations, first order partial differential equations, and second order partial differential equations, especially the Laplace equation. The last type of problem (considered in Part III) involves considerable difficulties, even in choosing a suitable formulation for it. It is necessary to consider problems of measure in function space, the definition of mean value of a functional, and the proper definition of the Laplace operator. Lévy's procedure is to consider these ideas in  $n$ -dimensional space, and then to proceed to the limit as  $n$  tends to infinity, so as to obtain a mean value for certain functionals defined in  $L_2$ . When the method of passage to the limit is chosen so that certain requirements are satisfied, other aspects of the geometrical situation become rather bizarre. Lévy has extensively revised Part III, but he states in his Preface that he still does not regard it as in definitive form. Choice of another method of passage from finite-dimensional space to the space  $C$  of continuous functions leads to the Wiener integral, which has other applications in functional analysis (cf. Paley and Wiener, *Fourier transforms in the complex domain*, Amer. Math. Soc. Colloquium Publications, vol. 19, 1934, Chaps. 9 and 10; also later memoirs by R. H. Cameron and W. T. Martin). It seems reasonable to forecast that investigation will reveal still other methods of defining integrals in the spaces  $C$  and  $L_2$ , which will have interesting applications.

A noteworthy change from the first edition is the addition of Part IV, by Franco Pellegrino, on analytic functionals of analytic functions. About forty percent of Part I has been omitted, including the old Chapter 3 on Lebesgue and Stieltjes integrals, Chapter 7 on orthogonal functions, and Chapter 8 on the equations of Fredholm and Volterra and integrodifferential equations. For a review of the first edition by C. A. Fischer, see Bull. Amer. Math. Soc. vol. 29 (1923) p. 229.

Part I is entitled *The Foundations of the functional calculus*. The