ON THE EXISTENCE OF PLANE CURVES WITH PRESCRIBED SINGULARITIES

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1. Introduction. It is the purpose of this lecture to present three problems on the existence of plane curves with prescribed singularities and to give some indication of the present state of these questions. The geometry is the classical algebraic geometry of the plane over the field of complex numbers.

If n, m, δ, κ, and i are the order, class, number of double points, number of cusps, and numbers of inflections of an irreducible plane curve $f=0$, these numbers are Pluecker characteristics of $f=0$, and satisfy Pluecker's equations:

$$\text{(P)} \qquad \begin{aligned} m &= n(n-1) - 2\delta - 3\kappa, \\ i &= 3n(n-2) - 6\delta - 8\kappa. \end{aligned}$$

It is natural to raise the question:

I. *Given a solution of* (P) *in non-negative integers, does there exist an irreducible plane curve for which these integers are the Pluecker characteristics?*

A solution of (P) with this geometrical interpretation will be said to be *proper*.

The second question arises in the theory of linear systems of plane curves. A *linear system* Σ is given by

$$\lambda_0 f_0 + \lambda_1 f_1 + \cdots + \lambda_d f_d = 0,$$

where the λ's are parameters and $f_0, f_1, \cdots, f_d$ are ternary forms of order x_0. If $f_0, f_1, \cdots, f_d$ are linearly independent, then d is the *dimension* of the linear system Σ. If P_i is a point such that all the curves of Σ are on P_i, then P_i is a base point of Σ. If the general curve of Σ has multiplicity x_i at P_i, then Σ is said to have multiplicity x_i at P_i. Let $P_1, P_2, \cdots, P_\rho$ be base points of Σ with multiplicities $x_1, x_2, \cdots, x_\rho$. If Σ contains all curves of order x_0 with these multiplicities at these points, then Σ is said to be *complete* (with respect to these multiplicities at these base points). If the linear conditions imposed by asking that a curve of order x_0 have these multiplicities at these points are independent, then Σ is said to be *regular* (with

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