THE ANALYTIC THEORY OF SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS[1]

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1. **Introduction.** The theory of ordinary linear differential equations in the complex domain has always attracted particular attention, and topics such as singularities of the regular type and equations of the Fuchsian class belong to those well-rounded gems for which the classical theory of functions of a complex variable is so justly famous.

Almost simultaneously with the fundamental researches of Fuchs and others on ordinary differential equations, efforts were made to develop a similar theory for systems of partial differential equations. Although mathematicians of the rank of Appell, Goursat, and Picard were among those interested in the matter, at the time these efforts were doomed to failure because of the insufficient understanding of functions of several complex variables, and also of some relevant topics in multiple series, algebraic geometry, topology, and groups. Today the problem could be attacked with greater hope of success, and the present survey seeks to prepare the ground for such an attack.

2. **Ordinary differential equations.** It will be useful to review briefly the theory of ordinary linear differential equations

$$\frac{d^n z}{dx^n} + g_1(x)\frac{d^{n-1}z}{dx^{n-1}} + \cdots + g_n(x)z = 0 \tag{1}$$

in the complex domain. The $g_i(x)$ are one-valued analytic functions of the complex variable x, and we shall assume that each $g_i(x)$ has at most a finite number of singularities. It is well known that such an equation has n linearly independent solutions, and that any $n+1$ solutions are linearly dependent. If all the $g_i(x)$ are regular in a neighborhood of $x=a$, then a is an *ordinary* point of the equation, and in a neighborhood of any such point the equation possesses a fundamental system of n analytic solutions represented by convergent power series. A singularity of any of the $g_i(x)$ is a *singular point* of the differential equation.

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