CLASSES OF TRANSFORMATIONS AND BORDERING TRANSFORMATIONS

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Introduction. I do not propose to give a complete or closed theory of any of the phases of the subjects I touch on in this address. In fact one of my primary purposes is to indicate what the natural unsolved problems are and what it would be reasonable to expect, and I shall raise as many questions as I answer. Many of my comments bear on results not yet published or on work in progress in connection with a book on fixed point theory. In a general way the first part of my remarks will interpret bordering transformations as those metrically near a canonical type, usually an isometry, while my later remarks will interpret bordering in a homotopy sense.

Our initial considerations will be devoted to the implications of what we may term uniform departures from standard transformations. The first to formulate some problems of this type was Ulam. In order to bring out their interconnections we present them as special cases or modifications of a general form and free the formulation from metric dependence although the metric cases alone have been treated. Suppose then that X and Y are uniform spaces with the vicinities of the diagonals denoted by $\mathfrak{N}$ and $\mathfrak{M}$. We assume the existence of a multiplication in these spaces denoted by $\bigcirc$ and $\oplus$. That is to say, $\bigcirc$ maps $X\times X$ into X, and $\oplus$ maps $Y\times Y$ into Y. Let T, S be transformations on X to Y satisfying

$$(1) \qquad (S(x_1 \bigcirc x_2),\ T_{x_1} \oplus T_{x_2}) \subset \mathfrak{M}.$$

We assume also a class of standard transformations from X to Y which we denote by U. We may then formulate a significant problem in the following way: Does there exist a standard transformation U and a vicinity $\mathfrak{N}'$ dependent on $\mathfrak{M}$ alone such that

$$(2) \qquad (Tx,\ Ux) \subset \mathfrak{N}',$$

and what is the "best" $\mathfrak{N}'$?

1. **Approximate additivity and isometry.** As illustration we can take for X a Banach space in its norm topology and for Y the real line. Let $S=T$ and let $\bigcirc$ and $\oplus$ be the respective vector addition

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