BOOK REVIEWS

Funktionentheorie. By C. Carathéodory. 2 vols. Basel, Birkhäuser, 1950.

The appearance of a book by a master of the stature of Carathéodory which is concerned for the most part with the classical aspects of a classical subject is an occasion of great interest. This is particularly so in the present case because it is reported that Carathéodory himself regarded his Funktionentheorie as his finest achievement. In apparaising this work it is well to recall that on the one hand, Carathéodory was a mathematician of very broad interests and on the other hand the theory of functions of a complex variable was an ever returning theme in his research.

It is therefore natural to anticipate that the present book would be written from a catholic point of view, that the treatment of general questions of convergence, continuity, and so on, would have a strong "real variable" flavor. This is indeed the case. Throughout the book there is constant reference to his Reelle Funktionen. One meets everywhere striking formulations of concepts which conventionally are phrased in other ways. For this reason the Funktionentheorie will be of considerable interest to the specialist who likes to compare notes.

Carathéodory's contributions to the theory of analytic functions are many and of lasting importance. We need call to mind only his early work on the Picard theorem which in turn led to the study of the coefficient problem for analytic functions with positive real part, the boundary behavior of conformal maps, the conformal mapping of variable regions, to mention but a few of his contributions. These interests are reflected in the second volume where the theory of bounded analytic functions, conformal mapping, the triangle functions and the Picard theorem are treated.

The author places great emphasis on the geometric point of view. The Weierstrassian aspects find less prominence than is often the case; instead the Riemannian aspect dominates. In fact, to lend point to this geometric tendency, some sixty pages of the beginning of the book are devoted to the geometry of the circle and non-euclidean geometry. In the introduction, Carathéodory says that he regards this chapter of geometry as the best entry to the theory of functions of a complex variable and he cites the role that these methods played in the achievements of Schwarz. In the treatment of this material synthetic and analytic methods are interwoven.

The first volume is divided into five parts. The first of these is concerned with the geometry of the circle, as we have just mentioned. The second part gives a brief resume of material on convergence, continuity, connectedness, the topology of the plane, line integrals.

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