BOOK REVIEWS

Leçons sur le calcul des coefficients d'une séries trigonométrique. Quatrieme Partie. *Les totalisations. Solution du problême de Fourier.* Premier Fascicule: *Les totalisations.* Deuxième Fascicule: *Appendices et tables générales.* By Arnaud Denjoy. Paris, Gauthier-Villars, 1949. Fasc. I, pp. 327–481, 1500 fr.; Fasc. II, pp. 483–715, 2200 fr.

These two books constitute the concluding part of a treatise, in four parts, devoted to the solution of one of the fundamental problems of trigonometrical series. The problem is to express trigonometrical series, that are everywhere convergent, in the Fourier form. This problem was successfully attacked in 1921 by Denjoy in five notes published in the Comptes Rendus de l'Academie des Sciences de Paris. In 1938, he gave a course of lectures on this topic at Harvard University, and these lectures are written up now in the form of a book.

The background to the problem is furnished by the uniqueness theory of trigonometrical series on the one hand and by the theory of integration of functions of a real variable on the other. If two trigonometrical series converge everywhere to the same sum, the series are identical; that is, the corresponding coefficients in the two series are equal. This is a theorem of Cantor (1872) which guarantees that if a function $f(x)$ has a trigonometric development

$$\frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

which converges everywhere, then this development is unique. If, in addition, the function $f(x)$ is Lebesgue integrable, then it is a Fourier development; in other words, the coefficients a_n, b_n are Fourier coefficients, related to $f(x)$ by means of the classical formulas of Fourier. This is a theorem of de la Vallée-Poussin (1912). The problem is to uphold this in the general case, without the restrictive assumption of Lebesgue integrability on $f(x)$. Denjoy solved this problem by evolving an elaborate process of *totalisation symmetrique à deux degrês* for the calculation of the coefficients. This process may be looked upon as a generalization of the notion of integration, and termed "trigonometric integration." It is more powerful than what is commonly known as the ordinary Denjoy integral which Denjoy had invented for the analogous, but somewhat less complicated, problem of proving that a function which is differentiable everywhere is the "integral"