

that "the problem of the infinitesimal stability of the periodic solutions of nonlinear systems always leads to a Hill equation." Following this thought, the author reduces the problem of stability to the analysis of the local behavior of the variational equations (of the Hill type) in the various parts of the response curve and applies this procedure to the Duffing equation. It turns out, however, that, on this basis, the free oscillations are unstable. Having ascertained this seemingly paradoxical result, the author ascribes it to the fact that the criterion of the "infinitesimal stability" (that is, the stability in the sense of the variational equations) ought to be replaced by that of the orbital stability. In fact, after a somewhat delicate argument, in §7, the author proves this point. In spite of this, the reader, particularly the beginner, must inevitably feel somewhat confused as to when to use one criterion and when to use the other. This question does not seem to find a definite answer in the text, probably because the author, as he said in his introduction, had to curtail considerably the theory of stability owing to lack of space. It seems, however, sufficiently simple to show that if the differential equations are referred to the "amplitude-phase" plane  $(a, \phi)$  instead of the usual  $(x, \dot{x})$  phase plane (namely,  $da/dt=f_1(a, \phi)$ ,  $d\phi/dt=f_2(a, \phi)$ ) the singular point  $f_1(a_0, \phi_0)=f_2(a_0, \phi_0)=0$  in this case represents the stationary periodic motion (if  $a_0 \neq 0$ ) and the variational equations ("the infinitesimal stability") give precisely the orbital stability in such a case, without any necessity of applying the theory of characteristic exponents of Poincaré. Reduction to this form is always possible if the differential equations do not contain time explicitly.

N. MINORSKY

*Differential algebra.* By Joseph Fels Ritt. (American Mathematical Society Colloquium Publications, vol. 33.) New York, American Mathematical Society, 1950. 8+181 pp. \$4.40.

It was a gigantic task that J. F. Ritt undertook twenty years ago: to give the classical theory of nonlinear differential equations a rigorous algebraic foundation. Emmy Noether and her school had done the same thing for the theory of algebraic equations and algebraic varieties, but differential equations are much more difficult than algebraic equations. Luckily, Ritt has gathered around himself a whole school of able collaborators: Raudenbusch, Strod, Kolchin, Howard Levi, Gourin, R. M. Cohn.

The present book is not just a revised and enlarged edition of the author's *Differential equations from the algebraic standpoint* (Colloquium Publications, vol. 14). It is written from a much higher point