

SELF-DUAL CONFIGURATIONS AND REGULAR GRAPHS

H. S. M. COXETER

1. Introduction. A configuration (m_c, n_d) is a set of m points and n lines in a plane, with d of the points on each line and c of the lines through each point; thus $cm = dn$. Those permutations which preserve incidences form a group, "the group of the configuration." If $m = n$, and consequently $c = d$, the group may include not only *symmetries* which permute the points among themselves but also *reciprocities* which interchange points and lines in accordance with the principle of duality. The configuration is then "self-dual," and its symbol (n_d, n_d) is conveniently abbreviated to n_d . We shall use the same symbol for the analogous concept of a configuration in three dimensions, consisting of n points lying by d 's in n planes, d through each point.

With any configuration we can associate a diagram called the *Menger graph* [13, p. 28],¹ in which the points are represented by dots or "nodes," two of which are joined by an arc or "branch" whenever the corresponding two points are on a line of the configuration. Unfortunately, however, it often happens that two different configurations have the same Menger graph. The present address is concerned with another kind of diagram, which represents the configuration uniquely. In this *Levi graph* [32, p. 5], we represent the points and lines (or planes) of the configuration by dots of two colors, say "red nodes" and "blue nodes," with the rule that *two nodes differently colored are joined whenever the corresponding elements of the configuration are incident*. (Two nodes of the same color are never joined.) Thus the Levi graph for (m_c, n_d) has m red nodes and n blue nodes, with each red node joined to c blue nodes and each blue node joined to d red nodes, so that there are $cm = dn$ branches altogether.

As simple instances in two dimensions we have the triangle 3_2 , whose Levi graph is a hexagon with red and blue vertices occurring

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¹ I take this opportunity to correct an error on the third line of page 31 of [13]: The words "the direct product . . . namely" should be deleted. This was kindly pointed out by Robert Frucht, whom I would also thank for his constructive criticism of the present address in manuscript. Numbers in brackets refer to the bibliography at the end of the paper.