NORMED LINEAR SPACES OF CONTINUOUS FUNCTIONS

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1. **Introduction.** In addition to its well known role in analysis, based on measure theory and integration, the study of the Banach space $B(X)$ of real bounded continuous functions on a topological space X seems to be motivated by two major objectives.

The first of these is the general question as to relations between the topological properties of X and the properties (algebraic, topological, metric) of $B(X)$ and its linear subspaces. The impetus to the study of this question has been given by various results which show that, under certain natural restrictions on X, the topological structure of X is completely determined by the structure of $B(X)$ [**3**; **16**; **7**],[1] and even by the structure of a certain type of subspace of $B(X)$ [**14**]. Beyond these foundational theorems, the results are as yet meager and exploratory. It would be exciting (but surprising) if some natural metric property of $B(X)$ were to lead to the unearthing of a new topological concept or theorem about X.

The second goal is to obtain information about the structure and classification of real Banach spaces. The hope in this direction is based on the fact that every Banach space is (equivalent to) a linear subspace of $B(X)$ [**1**] for some compact (that is, bicompact Hausdorff) X. Properties have been found which characterize the spaces $B(X)$ among all Banach spaces [**6**; **2**; **14**], and more generally, properties which characterize those Banach spaces which determine the topological structure of some compact or completely regular X [**14**; **15**]. These properties are defined in terms of concepts which are meaningful in all Banach spaces; in particular, no lattice [**10**] or ring [**8**; **9**; **11**] structure is presupposed.

I propose here to outline and supplement some recent results along the above two lines, using methods developed in [**14**]. In one or two instances details of proofs are given; these are brief and have not previously appeared in print.

2. **The mapping** $C(X)$. Let X be a completely regular topological space. The set of all real-valued, bounded continuous functions on X, with the usual laws of addition and multiplication by real num-

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[1] Numbers in brackets refer to the bibliography at the end of the paper.