SOME REMARKS ON RULED SURFACES

J. ERNEST WILKINS, JR.

In a previous paper [1][1] the author showed that the projective differential geometry of a nondevelopable ruled surface S in three-dimensional space could be studied by means of the expansion

$$
\begin{aligned}
z = xy &- \frac{1}{3}\gamma y^3 - \frac{1}{3}\gamma_u xy^3 - \frac{1}{12}\gamma_v y^4 - \frac{1}{6}\gamma_{uu}x^2y^3 - \frac{1}{12}\gamma_{uv}xy^4 \\
&+ \frac{1}{60}(10\gamma\gamma_u - \gamma_{vv})y^5 - \frac{1}{24}\gamma_{uuv}x^2y^4 \\
&+ \frac{1}{60}(10\gamma\gamma_{uu} + 10\gamma_u^2 - \gamma_{uvv})xy^5 \\
&+ \frac{1}{360}(20\gamma_u\gamma_v + 15\gamma\gamma_{uv} - \gamma_{vvv})y^6 + \cdots,
\end{aligned} \tag{1}
$$

for one nonhomogeneous coordinate z as a power series in the other two nonhomogeneous coordinates x and y. Here γ is a function of the form $A(v)u^2+B(v)u+C(v)$. It was also shown that there is a one-parameter family of cubic surfaces with fifth order contact with S, namely,

$$
\frac{1}{3}\gamma y^3 + Dz^3 + (z - xy)\left(Px - \frac{1}{4}(\gamma_v/\gamma)y + Mz + 1\right) + yz(Iy + Jz) = 0, \tag{2}
$$

where D is the parameter and P, M, I, and J are defined as follows:

$$
\begin{aligned}
P &= (15\gamma_v^2 + 40\gamma^2\gamma_u - 12\gamma\gamma_{vv})/80\gamma^3, \\
M &= (40\gamma^2\gamma_u\gamma_v + 12\gamma\gamma_v\gamma_{vv} - 80\gamma^3\gamma_{uv} - 15\gamma_v^3)/320\gamma^4, \\
I &= (5\gamma_v^2 + 40\gamma^2\gamma_u - 4\gamma\gamma_{vv})/80\gamma^2, \\
J &= (15\gamma_u\gamma_v^2 + 40\gamma^2\gamma_u^2 - 12\gamma\gamma_u\gamma_{vv} + 40\gamma^3\gamma_{uu})/240\gamma^3.
\end{aligned}
$$

In this paper we shall report some further results on nondevelopable ruled surfaces which can be obtained with the help of the above formulas.

It was shown in [1] that S is a cubic surface if and only if $\mathcal{A} = \mathcal{B} = 0$, where

Received by the editors October 7, 1947, and in revised form September 30, 1948.

[1] Numbers in brackets refer to the bibliography at the end of the paper.