SOME ELEMENTARY TOPOLOGICAL PROPERTIES OF ESSENTIAL MAXIMAL MODEL CONTINUA

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1. **Introduction.** Let T: $z=t(w)$, $w\in D$, be a bounded continuous transformation from a bounded connected open set D in the w-plane into the z-plane. Radó and Reichelderfer [3, p. 263][2] have defined essential maximal model continua $\gamma(z)$ for such transformations. If A is a set in the z-plane, let $E(A)$ be the point set sum of the e.m.m.c. $\gamma(z)$ for $z\in A$. Thus $E(A)$ is a subset of D. We are primarily interested in the topological properties of $E(A)$. Since most of the reasoning is valid for more general spaces, it is carried out first for these spaces and then in §6 the results are applied to the above situation. These results are also applicable to the set $E^+(A)$ and $E^-(A)$ defined by Reichelderfer in [4].

2. **Preliminaries.** Let T: $y=t(x)$, $x\in X$, be a single-valued continuous transformation from a normal Hausdorff space X into a topological space Y (these terms are used in the sense of [1, chap. 1]). Suppose that for each set $A\subset Y$ there has been defined a set $E(A)\subset X$ such that

$$E(A) = \sum_{y\in A} E(y), \tag{1}$$

$$E(y) = \sum_{\gamma\in S(y)} \gamma, \tag{2}$$

where $S(y)$ is a class of disjoint nonempty closed connected sets $\gamma\subset X$ such that $\gamma\subset T^{-1}(y)$ and having the following property:

P. If $\gamma_0\in S(y_0)$ and if G is any open set such that $\gamma_0\subset G$, then there exists a neighborhood $N(y_0)$ such that if $y\in N(y_0)$ then there exists a $\gamma\in S(y)$ such that $\gamma\subset G$.

Let $f(y)$ be the number of $\gamma\in S(y)$. Thus $f(y)=0, 1, 2, \cdots$, or $+\infty$, no distinction being made between infinite cardinals.

3. **Closed sets** A.

THEOREM 1. *Let T, A, $E(A)$, and $f(y)$ be given as in §2. If A is closed and if $f(y)$, $y\in A$, is upper semi-continuous and bounded on A, then $E(A)$ is closed.*

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[2] Numbers in brackets refer to the bibliography at the end of the paper.