THE UNIQUENESS OF A CERTAIN LINE INVOLUTION

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In a forthcoming paper [1][1] on the classification of line involutions we pointed out that any line involution in S_3 with the property that a general line and its image do not intersect has associated with it, besides its order, m, and the order, i, of its invariant complex, two other important numerical characteristics which we called k and n. The first of these, k, is the number of lines of an arbitrary pencil which meet their images. Recalling the well known mapping relation between the lines of S_3 and the points of a nonsingular V_4^2 in S_5, the second, n, is the order of the ruled surface formed by the rays which join the points of a general line of V_4^2 to their respective images. The four quantities, m, i, k, n, are not independent but satisfy the following relations [1]

$$\text{(A)} \qquad n - i = k, \qquad n + i = m + 1.$$

It is the purpose of the present note to establish the fact that there is a unique involution associated with the set $m=n=2$, $i=k=1$, namely the involution whose point equivalent on V_4^2 is defined by the transversals of a general line, λ, of S_5 and an S_3 contained in the S_4 which is tangent to V_4^2 at one of the two intersections of λ and V_4^2 [2, 3]. In constructing our proof we shall find it convenient to work exclusively in S_5 rather than in the three-dimensional space where the line involution is actually defined.

We shall begin by establishing the fact that the surface which is the image of a general plane of V_4^2 is of the second order. This cannot be inferred as obvious from the fact that the order of the transformation is two, because one may show by example that far from being self-evident, it is not even true that the order of the image of a plane of V_4^2 is always the order of the involution. Moreover it is not true that planes of opposite families on V_4^2 necessarily have images of the same order. For instance in a paper in this Bulletin [4] Clarkson discussed a simple involution of order two in which bundles of lines are transformed into (3, 1) congruences while plane fields of lines are transformed into (1, 1) congruences. In other words, on V_4^2 an ω-plane is transformed into a surface of the fourth order, while a ρ-plane is transformed into a quadric surface.

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[1] Numbers in brackets refer to the bibliography at the end of the paper.