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A NOTE ON LEAST COMMON LEFT MULTIPLES

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1. **Introduction.** Consider n-by-n matrices $A, B, \cdots$ with elements in a principal ideal ring and recall the following definitions. If $A = BC$, then A is a *left multiple* of C and C is a *right divisor* of A. If $A = RD$ and $B = PD$, then D is a *common right divisor* of A and B; if, furthermore, D is a left multiple of every common right divisor of A and B, then D is a *greatest common right divisor* of A and B. If $M = PA = QB$, then M is a *common left multiple* of A and B; if, furthermore, M is a right divisor of every common left multiple of A and B, then M is a *least common left multiple* of A and B. If $FE = I$, where I is the identity matrix, then E is a *unimodular* matrix. If E is unimodular, then EA is a *left associate* of A.

The basic tool in the following constructions is the theorem[1] that any given matrix A is the left associate of a uniquely determined matrix A_1, known as the Hermite canonical triangular form, having zeros above the main diagonal, having elements below the main diagonal in a prescribed residue class modulo the diagonal element above, having each diagonal element in a prescribed system of non-associates, and if a diagonal element is zero, having the corresponding row all zero.

C. C. MacDuffee has presented the following method,[2] due in essence to E. Cahen and A. Chatelet, for finding a greatest common

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[1] C. C. MacDuffee, *Matrices with elements in a principal ideal ring*, Bull. Amer. Math. Soc. vol. 39 (1933) pp. 570–573.

[2] C. C. MacDuffee, loc. cit. p. 573.