SPECIAL PROPERTIES OF MEASURE PRESERVING TRANSFORMATIONS

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1. **Summary.** In studying problems[1] concerned with the qualitative description of bounded trajectories, in a region free of singular points, associated with a flow

$$dx_i/dt = P_i(x_1, \cdots, x_n) \qquad (i = 1, \cdots, n) \tag{1.1}$$

the P_i being holomorphic in the x_i, we considered the possibility of finding "point conditions" on the P_i which would insure a smooth behavior on the part of a trajectory—or more specifically, on any motion in its limit set. For example, the restriction that the transformation defined by equations (1.1) preserve n measure is expressible by the point condition

$$\sum_{i=1}^{n} \frac{\partial P_i}{\partial x_i} = 0.$$

Being unable to ensure the behavior desired by this condition, we sought stronger conditions. In particular we asked, "What are the conditions on the P_i such that the transformations defined by (1.1) preserves p measure where p is restricted to values $1 \leqq p \leqq n-1$?" The answer to this question and also to the question, "Is this a restrictive condition?" is contained in the following theorem.

THEOREM I. *The condition that the flow defined by*

$$dx_i/dt = P_i(x_1, \cdots, x_n) \qquad (i = 1, \cdots, n),$$

the P_i being holomorphic, preserve p measure, p any (fixed) integer between 1 *and $n-1$, is that*

$$\partial P_i/\partial x_j = -\partial P_j/\partial x_i$$

for all i and j. These conditions imply that the motion is rigid.

It is an open and apparently difficult question as to whether every point transformation (we are considering only homeomorphisms)—of a sufficiently differentiable class—of E^3 onto itself is obtainable from a flow, that is, from the solutions of a system of first order equations of the type (1.1). (In E^2 there are point transformations which are *not* embeddable in flows.) Thus it is natural to ask for how

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[1] *On systems of ordinary differential equations*, which will appear elsewhere.