NOTE ON HOMOGENEOUS PLANE CONTINUA

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In his Doctoral Dissertation (Texas, 1947), E. E. Moise proved that there exists a compact plane continuum M (not an arc) which is homeomorphic to each of its subcontinua [1].[1] Subsequently, R. H. Bing showed that M is homogeneous [2]. Bing's result flatly contradicts the previously announced result of G. Choquet to the effect that a homogeneous, compact, plane continuum must be a simple closed curve [3]. It is the purpose of this note to show that had Choquet assumed in addition to homogeneity that the continuum was aposyndetic[2] at some point, or that some point of the continuum failed to be a weak cut point[3] of it, then his conclusion would have been valid.

THEOREM 1. *If a compact, plane continuum M is both homogeneous and aposyndetic, then M is a simple closed curve.*

PROOF. If a point of M is of order 2 in M, then M is a simple closed curve [4]. So suppose that *no* point of M is of order 2 in M.

Let G denote the collection of all the complementary domains of M. Because M is homogeneous and contains a non-separating point, no point of M separates M. Since M is aposyndetic, M is semi-locally-connected [5]. Hence each element of G is a simple domain [6, 7]. Let the simple closed curve J denote the boundary of an element D of G.

Case 1. If $M-J$ is connected, then each point of M belongs to some such simple closed curve lying in M. Consequently each point of M belongs to the boundary of an element of G. But G is countable. Hence $M=\sum J_i$ $(i=1, 2, 3, \cdots)$, such that for each i, J_i is the boundary of an element of G. Since no point of M is of order 2 in M, each point of J_i is a limit point of $M-J_i$. This contradicts a well known theorem (Baire).

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[1] Numbers in brackets refer to the bibliography at the end of this paper.

[2] A continuum M is said to be *aposyndetic* at a point x of M if for each point y of $M-x$, there exists a subcontinuum of M and an open subset U of M such that $M-y\supset H\supset U\supset x$. If a continuum is aposyndetic at each of its points, then it is said to be aposyndetic.

[3] A point p of a continuum M is a *weak cut point* of M if $M-p$ is not strongly (continuum-wise) connected. For other definitions the reader is referred to Moore's book or Whyburn's book, volumes 13 and 28, respectively, of the American Mathematical Society Colloquium Publications.