$$C_n = \bigcup_{j=0}^{n} (\text{closure } R_j) \cup \{\mathcal{S} \sim \bigcup_{j=0}^{\infty} (\text{closure } R_j)\}.$$

Clearly, for each integer $n$,

$$\bigcup_{j=0}^{\infty} C_j = \mathcal{S}. \qquad C_n \subset C_{n+1} \in F,$$

After checking the hereditariness of $F$, we infer from 4.2 that each open set is $\phi$ measurable $F$. Hence, if we recall 3.5, $C_n$ is $\phi$ measurable $F$ for each integer $n$. Thus $F$ is $\phi$ convenient. Reference to 4.3 completes the proof.

UNIVERSITY OF CALIFORNIA

---

# ON THE DISTRIBUTION OF THE VALUES OF $|f(z)|$ IN THE UNIT CIRCLE

ROBERT BREUSCH

1. **Summary.** Let $f(z)=1+a_1z+\cdots$ be analytic for $|z|\leqq 1$, $f(z)\not\equiv 1$. Then $|f(z)|$ will be greater than 1 at some points of the unit circle, and less than 1 at others. Calling $A(f)$ the area of the set of points within the unit circle, for which $|f(z)|\geqq 1$, let $\alpha$ and $\beta$ be the two largest non-negative constants such that $\alpha\leqq A(f)\leqq\pi-\beta$, for every $f(z)$. It is shown that $\alpha=\beta=0$; in other words, if $\epsilon$ is arbitrarily small positive, there are functions $f(z)$ such that $A(f)<\epsilon$, and others such that $A(f)>\pi-\epsilon$. The same is true, if $f(z)$ is restricted to polynomials $\prod_{\nu=1}^{n}(z-z_\nu)$ with $\prod_{\nu=1}^{n}|z_\nu|=1$. These statements will be proved in §2. §3 contains a few additional results, given without proofs.

2. **Proofs.** The statements made in the summary are contained in the following theorem.

THEOREM. *Let $P$ stand for the set of polynomials over the complex field of the form $f(z)=\prod_{\nu=1}^{n}(z-z_\nu)$, with $\prod_{\nu=1}^{n}|z_\nu|=1$; let $A(f)$ denote the area of the set of points in the unit circle, for which $|f(z)|\geqq 1$; let $\epsilon$ be an arbitrarily small positive number. Then $P$ contains polynomials $f_1(z)$ and $f_2(z)$ such that $A(f_1)>\pi-\epsilon$, and $A(f_2)<\epsilon$.*

---

Presented to the Society, December 31, 1947; received by the editors January 7, 1948.