If $\xi_{\nu} \geqq 1$ for each $\nu$, then by reasoning analogous to that of the preceding example, it may be shown, for any set (a), that there is no point $p$ such that $t<p$ implies that $\log S_{t}(a, \xi)$ is convex nor a point $p$ such that $t>p$ implies that $\log S_{t}(a, \xi)$ is concave. Hence Theorem 4 applies to all such functions $\log S_{t}(a, \xi)$. However, for this case the conclusion of the general theorem is weaker than the known result that $\log S_{t}(a, \xi)$ is convex for all positive $t$ and concave for all negative $t .^{2}$

University of California at Los Angeles

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## HOMOMORPHISMS ON BANACH SPACES

## M. E. MUNROE

1. Introduction. Let $E$ be a Banach space and $E^{*}$ its conjugate space. Let $G$ be a closed linear subspace of $E$, and let $\Gamma=\left\{f \mid f \in E^{*}\right.$, $f(x)=0$ for $x \in G\}$. Krein and Šmulian have shown [4, Theorem 12'] ${ }^{1}$ that $G^{*}=E^{*} / \Gamma$ in the sense that the two spaces are algebraically isomorphic and that the usual definitions of norm in the two are equivalent. Noting the algebraic isomorphism, let us look at the topological aspects of this equivalence in a slightly different light. $G^{*}$ being a factor space of $E^{*}$, there is defined a natural homomorphism [5, p. 64] $T\left(E^{*}\right)=G^{*}$. Since they are using the induced topology [5, p. 58] in $E^{*} / \Gamma$, Krein and Smulian prove what is equivalent to the theorem that the transformation $T$ is continuous and open (see [5, Theorem 12]). Stated in this way, incidentally, the result follows immediately from the Hahn-Banach theorem by means of the usual neighborhood argument for continuity and openness.

However, the homomorphism $T\left(E^{*}\right)=G^{*}$ suggests other topological questions the answers to which are not quite so obvious. Specifically, what are the topological properties of $T$ when $E^{*}$ and $G^{*}$ are given topologies other than their norm topologies?

The conjugate to a Banach space may be topologized in any one of several well known ways. The most common such topologies are the norm, weak, weak*, bounded weak and bounded weak*. We shall

[^1]
[^0]:    ${ }^{2}$ See Beckenbach, An inequality of Jensen, Amer. Math. Monthly vol. 53 (1946) pp. 501-505.

[^1]:    Presented to the Society, October 25, 1947; received by the editors September 10, 1947.
    ${ }^{1}$ Numbers in brackets refer to the bibliography at the end of the paper.

