BOUNDS ON CHARACTERISTIC VALUES

EDWARD W. BARANKIN

In a previous paper [1][1] the author obtained several upper bounds for the characteristic roots of a finite matrix. The purpose of this note is to establish the corresponding results for characteristic values of certain linear transformations in certain infinite spaces.

1. **Infinite matrices.** We consider a non-null infinite matrix, $A=(a_{st})$, s, $t=1, 2, \cdots$, of complex numbers. (The present considerations pertain also to the finite case when $a_{st}=0$ for $s>n$ and $t>n$.) We shall say that A is *absolutely summable* (abbreviated a.s.) when the double series $\sum_{s,t}|a_{st}|$ converges. In general, if $\sum_{s,t}|a_{st}|^p$ converges for a positive number p, A will be said to be *absolutely summable* (p). If A is a.s.(p), then $\sum_t|a_{st}|^p$ converges for all s, $\sum_s|a_{st}|^p$ converges for all t, and

$$\sum_{s,t}|a_{st}|^p = \sum_s\sum_t|a_{st}|^p = \sum_t\sum_s|a_{st}|^p.$$

We define

$$R_s^{(p)} = \sum_t |a_{st}|^p, \qquad T_t^{(p)} = \sum_s |a_{st}|^p,$$

and, for brevity, $R_s=R_s^{(1)}$, $T_t=T_t^{(1)}$. Clearly if A is a.s.(p), it is a.s.(q) for every $q>p$. Observe also that $R_s^{(p)}=0$ or $T_t^{(p)}=0$ imply, respectively, the vanishing of all elements in the sth row or of all elements in the tth column of A.

Let A be a.s.(p), $p\leqq 1$, and l denote the space of bounded sequences of complex numbers $\{x_r\}$. Throughout, we shall view A as a transformation in l. As such, A carries l into a proper sub-manifold. For, let $\{x_r\}\in l$, and $|x_r|<M$, $r=1, 2, \cdots$. Then the transform of $\{x_r\}$ has the components

$$y_s = \sum_t a_{st}x_t, \qquad s = 1, 2, \cdots.$$

Since $p\leqq 1$, we have

$$|y_s|^p \leqq \left(\sum_t |a_{st}|\cdot|x_t|\right)^p \leqq M^p\left(\sum_t|a_{st}|\right)^p \leqq M^p\sum_t|a_{st}|^p,$$

Received by the editors August 17, 1946, and, in revised form, September 5, 1947.

[1] Numbers in brackets refer to the bibliography at the end of the paper.