NONASSOCIATIVE VALUATIONS

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In a paper entitled *Noncommutative valuations*, Schilling [8][1] proved that if an algebra of finite order over its center is relatively complete in a valuation (where the value group of the nonarchimedean, exponential valuation is not assumed to be commutative) then the value group is commutative. A similar type of theorem, proved by Albert [1], states that if an algebra of finite order over a field is an ordered algebra, then the algebra is itself commutative.

In the present paper, the notions of valuation and ordered ring are carried over in the obvious fashion to the case of nonassociative algebras (the set of values of a valuation are no longer required to lie in an ordered group, but only in an ordered loop). In this situation, an analogue of Schilling's result remains true with an added hypothesis: If an algebra of finite order has a unity quantity and has a valuation inducing a rank one valuation of the base field, then the value loop of the algebra is commutative, associative, and archimedean-ordered. No completeness is needed. In particular, every valuation of an algebra (with a unity quantity) of finite order over an algebraic number field has a group of real numbers for its value loop.

However, in general the obvious extensions of both Schilling's and Albert's results are false, since there are noncommutative, nonassociative algebras of arbitrary finite order over a field which have valuations with nonassociative value loops and which are ordered algebras. Examples of such algebras are obtained by proving that a necessary and sufficient condition for an ordered loop L to be the value loop of some algebra of finite order is that some subgroup of the center of L have finite index in L. We construct some such loops in §3. All such loops will be determined in another paper.

1. **Ordered loops.** An ordered loop L is a set of elements $x, y, z, \cdots$ on which are defined a binary function, $+$, and a binary relation, $>$, with the following properties:

(1) $+$ is a single-valued function on LL to L.

(2) If x and y are in L, then there exist unique elements u and v in L such that $u+x=y$ and $x+v=y$.

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[1] Numbers in brackets refer to the bibliography at the end of this paper.