THE ROLE OF THE CENTER IN THE THEORY OF DIRECT DECOMPOSITIONS

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It has been noticed for a long time that the center plays a fundamental part in the theory of direct decompositions of operator groups and loops. In particular it has been found that the existence of isomorphic refinements of direct decompositions can be assured by imposing conditions which refer solely to the center.[1] It is the object of the present note to give an explanation for these phenomena by proving quite generally that the validity of the refinement theory in an operator loop is a consequence of the validity of this theory in the center.[2]

In our discussion of operator loops and their direct decompositions we shall use the notations and fundamental definitions which we introduced in a previous paper (Baer [1]) and which are, of course, quite analogous to those customarily used in the theory of operator groups. The refinement theory, however, which we developed recently differs materially from previous statements of the theory, and thus we restate it here in the form best suited to our present purposes.

DEFINITION. *If*

$$L = A \oplus B = D \oplus E \tag{i}$$

are direct decompositions of the M-loop L into direct sums of two M-subloops, if the refinements

$$A = A' \oplus A'', \quad B = B' \oplus B'', \quad D = D' \oplus D'', \quad E = E' \oplus E'' \tag{ii}$$

satisfy the conditions

$$\begin{aligned} A' \oplus B' &= B' \oplus D' = D' \oplus E' = E' \oplus A' (= L'), \\ A'' \oplus D'' &= D'' \oplus E'' = E'' \oplus B'' = B'' \oplus A''(= L''), \end{aligned} \tag{iii}$$

then the decompositions (ii) *constitute canonical refinements of the pair* (i) *of direct decompositions of L.*

Using this definition our refinement theory may be expressed briefly in the form of the following proposition.

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[1] See Baer [1] for bibliographical references and a survey of the pertinent facts, in particular the theorems of Kořínek and Kurosh. Numbers in brackets refer to the bibliography at the end of the paper.

[2] A more precise formulation of this statement will be given immediately below.