NONLINEAR NETWORKS. IIb

R. J. DUFFIN

This note is concerned with the quasi-linear properties of an n-dimensional transformation

$$
(1)\qquad
\begin{aligned}
y_1 &= S_1(x_1, \cdots, x_n),\\
&\cdots\cdots\cdots\cdots,\\
y_n &= S_n(x_1, \cdots, x_n).
\end{aligned}
$$

More precisely it is shown by imposing certain conditions A on the functions S_i that the transformation has the property of possessing a unique inverse. In this property, then, the transformation is analogous to a nonsingular linear transformation. The actual conditions A imposed on the functions S_i are so chosen that the transformation (1) shall be a generalization of the equations which define the steady flow of current in electrical networks made up of quasi-linear conductors. It is reasonable to believe, however, that the methods developed here can, with suitable modification, be used to study the quasi-linear properties of types of transformations which have nothing to do with electrical networks.

At least three different methods of attack are available: The first method is to set up a form (analogous to a positive definite quadratic form) such that the equations (1) are the conditions that this form take on its minimum value. This insures the existence of a solution. A second positive definite form involving the differences of two transformations proves the uniqueness. It is well known that for linear transformations this method has the disadvantage of being applicable only for self-adjoint transformations. A similar disadvantage appears in the nonlinear case. This method was exploited in two previous notes: *Nonlinear networks*. I, and *Nonlinear networks*. IIa. (No appeal is made in this note to results obtained in the previous notes.)

The second method, which is the one employed in this note, is to impose conditions A_1 such that the Brouwer fixed point theorem is available. This insures the existence of the inverse transformation. Corresponding to the differences of two transformations, we associate a linear transformation somewhat analogous to a differential transformation. Conditions A_2 are then imposed, which insure that the associated linear transformation satisfies A_1. Hence, the linear trans-

Presented to the Society, September 15, 1945; received by the editors April 21, 1947.