CONSTRAINED MOTION UPON A SURFACE UNDER A GENERALIZED FIELD OF FORCE

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1. **Introduction.** We shall present here some theorems concerning the trajectories of a particle moving in a given smooth surface when acted upon only by a generalized field of force and free of the pressure of the surface.

Kasner has developed the differential geometry of the dynamical trajectories of general positional fields of force on a plane, in space, and on a surface, in his Princeton Colloquium Lectures.[1] Recently Kasner and DeCicco have introduced the concept of *generalized* fields of force which depend not only upon the position of the point but also upon the direction through the point. Positional fields may be described as *isotropic* and generalized fields as *anisotropic*.[2]

Consider a generalized field of force defined in a certain region of ordinary space.[3] Let Σ be a smooth surface in this particular region. The trajectories described by a particle constrained to move in Σ and acted upon only by this generalized field of force are ∞^3 in number, just as is the case for a plane.

In general curvilinear coordinates (u, v) of Σ, these trajectories are defined by a differential equation of the form

(F) $$v''' = F(u, v, v') + G(u, v, v')v'' + H(u, v, v')v''^2.$$

Conversely any differential equation of this form which is satisfied by the totality of geodesics of Σ represents the trajectories of a particle constrained to move in Σ, which is acted upon only by a generalized field of force.

Kasner has proved that in the (u, v)-plane, the curves defined by a differential equation of third order of the form (F) may be characterized in the following manner.[4] The locus of the foci of the osculating parabolas constructed to the ∞^1 trajectories passing

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[1] Kasner, *Differential-geometric aspects of dynamics*, Amer. Math. Soc. Colloquium Publications, vol. 3, 1913, 1934 (referred to as the Princeton Colloquium). Also see series of papers in Trans. Amer. Math. Soc. vols. 7–11 (1906–1910).

[2] Kasner and DeCicco, *A generalized theory of dynamical trajectories*, Trans. Amer. Math. Soc. vol. 54 (1943) pp. 23–38.

[3] Kasner and DeCicco, *Generalized dynamical trajectories in space*, Duke Math. J. vol. 4 (1943) pp. 733–742.

[4] See the Princeton Colloquium, pp. 104–107.