A NOTE ON THE SCHMIDT-REMAK THEOREM

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Let G be a group with operator domain Ω. We shall say that G satisfies the modified maximal condition for Ω-subgroups if the chain $H_1 \subset H_2 \subset \cdots \subset H \neq G$ is finite whenever $H_1, H_2, \cdots, H$ are Ω-subgroups of G.

Let $A_1, A_2, \cdots$ be a countable set of groups. The direct product of $A_1, A_2, \cdots$ will be defined to be the set of elements $(a_1, a_2, \cdots)$ where a_i is an element of A_i for $i = 1, 2, \cdots$, and where but a finite number of the a_i are not the identity elements of the groups in which they lie. A product in the group is defined by the usual componentwise composition of two elements. This group will have the symbol $A_1 \times A_2 \times \cdots$.

The following theorem is in a sense a generalization of the Schmidt-Remak theorem.

THEOREM. *Let G be a group with operator domain Ω, and let Ω contain the inner automorphisms of G. Let $G = A_1 \times A_2 \times \cdots$ where each of the Ω-subgroups A_i is directly indecomposable, and each satisfies the minimal condition and the modified maximal condition for Ω-subgroups. Then if $G = B_1 \times B_2 \times \cdots$ is a second direct product decomposition of G into indecomposable factors, the number of factors will be the same as the number of the A_i. Further the A_i may be so rearranged that $A_i \cong B_i$, and for any j*

$$G = B_1 \times B_2 \times \cdots \times B_j \times A_{j+1} \times A_{j+2} \times \cdots .$$

A proof of the theorem can be based on any standard proof of the Schmidt-Remak theorem such as that given by Jacobson[1] or by Zassenhaus[2] with but slight changes in the two fundamental lemmas.

We state the following lemmas for a group G with operator domain Ω, and we assume that for G and Ω:

(1) Ω contains all inner automorphisms of G.

(2) G satisfies the minimal condition and the modified maximal condition for Ω-subgroups.

(3) G is indecomposable into the direct product of Ω-subgroups.

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[1] Nathan Jacobson, *The theory of rings*, Mathematical Surveys, vol. 2, New York, 1943.

[2] H. Zassenhaus, *Lehrbuch der Gruppentheorie*, Leipzig, 1937.