

an even entire function of order one and minimal type. A counter example shows that the word "minimal" cannot be replaced by "normal." (Received June 27, 1947.)

#### APPLIED MATHEMATICS

333. H. E. Salzer: *Checking and interpolation of functions tabulated at certain irregular logarithmic intervals.*

For functions that are usually represented upon semi-logarithmic graph paper, that is, which behave as polynomials in  $\log x$ , the problem of checking or interpolation when the  $x$ 's are in geometric progression is quite simple due to the uniform interval in  $\log x$ . But in practice functions are often given at some or all of the points 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000 (same as .001, .002, .005, and so on, or .01, .02, and so on). In the present paper coefficients are given which facilitate: (I) checking of such functions when given at some of the more frequently occurring combinations of those points, by obtaining the last divided difference; (II) Lagrangian interpolation according to a generalization of the scheme recently given by W. J. Taylor, *Journal of Research, National Bureau of Standards*, vol. 35 (1945) pp. 151-155, RP 1667. (Received July 16, 1947.)

334. H. E. Salzer: *Coefficients for expressing the first twenty-four powers in terms of the Legendre polynomials.*

Exact values of the coefficients of  $P_m(x)$ , the  $m$ th Legendre polynomial, in the expression for  $x^n$  as a series of Legendre polynomials are tabulated for  $n=0, 1, 2, \dots, 24$ . Previous tables due to Byerly or Hobson do not extend beyond  $n=8$ , and are inadequate for many needs. These coefficients will be useful in approximating a polynomial of high degree (denoted by  $f(x)$  after normalization to the interval  $[-1, 1]$ ) by a polynomial of lower degree which will be best in a well known least square sense; that is, for a preassigned  $r$ , they will be used to obtain the polynomial  $q_r(x)$  of degree not greater than  $r$  which minimizes  $\int_{-1}^1 [f(x) - q_r(x)]^2 dx$ . (Received July 3, 1947.)

335. H. E. Salzer: *Complex interpolation over a square grid, based upon five, six, and seven points.*

Lagrangian coefficients are tabulated for complex interpolation of an analytic function of  $Z=x+iy$ , which is given over a square grid in the  $Z$ -plane. The formulas employed here are based upon the values of the function at five, six, or seven points which are chosen so as to be as close together as possible, at the sacrifice of possible symmetry. (This is a continuation of the tables contained in the article by A. N. Lowan and H. E. Salzer, *Coefficients for interpolation within a square grid in the complex plane*, *Journal of Mathematics and Physics* vol. 23 (1944), which gives the coefficients for the 3- and 4-point cases.) Denoting the reference point in the lower left-hand corner by  $Z_0$ , and  $h$  the length of the grid, so that  $Z=Z_0+Ph$ , where  $P=p+iq$ , the approximating  $n$ -point formulas are of the well known form  $\sum L_j^{(n)}(P)f(Z_j)$ . Exact values of the coefficients  $L_j^{(n)}(P)$  are given for  $p=0, .1, .2, \dots, 1.0$  and  $q=0, .1, .2, \dots, 1.0$ . A method for inverse interpolation is indicated, based upon the coefficients of  $P^m$  in  $L^{(n)}(P)$ . (Received July 16, 1947.)