# ON THE CHARACTERISTIC EQUATIONS OF CERTAIN MATRICES 

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In a paper to be published soon in the Annals of Mathematical Statistics, R. v. Mises obtains the following theorem on matrices from results in the theory of probability.

Theorem. Let $A=\left(a_{\kappa \lambda}\right), B=\left(b_{\kappa \lambda}\right)$, and $C=\left(c_{\kappa \lambda}\right)$ be square matrices of order $n$. If the elements of $A$ and $C$ satisfy the conditions

$$
\begin{array}{lr}
r_{\kappa}=\sum_{\nu=1}^{n} a_{\kappa \nu}=0 & (\kappa=1,2, \cdots, n), \\
s_{\lambda}=\sum_{\nu=1}^{n} a_{\nu \lambda}=0 & (\lambda=1,2, \cdots, n) \\
c_{\kappa \lambda}=c_{\kappa}+c_{\lambda} & (\kappa, \lambda=1,2, \cdots, n) \tag{3}
\end{array}
$$

where $c_{1}, c_{2}, \cdots, c_{n}$ are arbitrary numbers, then the matrices $A B$ and $A(B+C)$ have the same characteristic equation.

In the following a purely algebraic proof of this theorem will be given.

Proof. We set

$$
\sum_{\nu=1}^{n} a_{k} c_{\nu}=q_{k} \quad(\kappa=1,2, \cdots, n) .
$$

Then we have by (1) and (3)
(4)

$$
\begin{aligned}
& \left.A C=\left(\begin{array}{cccc}
a_{11} & a_{12} & \cdots & a_{1 n} \\
a_{21} & a_{22} & \cdots & a_{2 n} \\
\cdots & \cdots & \cdots & \cdot \\
a_{n 1} & a_{n 2} & \cdots & a_{n n}
\end{array}\right)\left(\begin{array}{ccc}
c_{1}+c_{1} & c_{1}+c_{2} & \cdots \\
c_{1}+c_{n} \\
c_{2}+c_{1} & c_{2}+c_{2} & \cdots \\
c_{2}+c_{n} \\
\cdots & \cdots & \cdots
\end{array}\right) \cdot \cdots \cdot \cdot\right\} \\
& =\left(\begin{array}{cc}
q_{1}+c_{1} r_{1} & q_{1}+c_{2} r_{1} \cdots q_{1}+c_{n} r_{1} \\
q_{2}+c_{1} r_{2} & q_{2}+c_{2} r_{2} \cdots q_{2}+c_{n} r_{2} \\
\cdots \cdots \cdot & \cdots \cdots \cdots \\
q_{v}+c_{1} r_{n} & q_{n}+c_{2} r_{n} \cdots q_{n}+c_{n} r_{n}
\end{array}\right)=\left(\begin{array}{ccc}
q_{1} & q_{1} \cdots q_{1} \\
q_{2} & q_{2} \cdots & q_{2} \\
\cdots & \cdots & \cdot \\
q_{n} & q_{n} \cdots q_{n}
\end{array}\right) .
\end{aligned}
$$

Let $P$ be the triangular matrix
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