of Lemma 3 to obtain suitable θ's for groups of the form $Z_1 \times Z_2 \times Z_3$ where Z_i are cyclic of order 2^{n_i}. However, it should be noted that if $G \cong G_1 \times G_2$, a one-to-one mapping θ of G upon G may be defined by

$$\theta[(x, y)] = [\theta_1(x), \theta_2(y)]$$

where θ_1 and θ_2 are one-to-one mappings of G_1 upon G_1 and G_2 upon G_2 respectively. Moreover θ satisfies the relationship $O(\eta) \geqq O(\eta_1) \cdot O(\eta_2)$. Thus if $O(\eta_1) = n(G_1)$, $O(\eta_2) = n(G_2)$ we would have $O(\eta) = n(G_1 \times G_2)$ and θ is represented explicitly in terms of θ_1 and θ_2.

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ON RINGS WHOSE ASSOCIATED LIE RINGS ARE NILPOTENT

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1. **Introduction.** With any ring R we may associate a Lie ring $(R)_l$ by combining the elements of R under addition and commutation, where the commutator $x \circ y$ of two elements $x, y \in R$ is defined by

$$x \circ y = xy - yx.$$

We call $(R)_l$ the Lie ring associated with R, and denote it by $\mathfrak{R}$. The question of how far the properties of $\mathfrak{R}$ determine those of R is of considerable interest, and has been studied extensively for the case when R is an algebra, but little is known of the situation in general. In an earlier paper the author investigated the effect of the nilpotency of $\mathfrak{R}$ upon the structure of R if R contains a nilpotent ideal N such that R/N is commutative.[1] In the present note we prove that, for an arbitrary ring R, the nilpotency of $\mathfrak{R}$ implies that the commutators of R of the form $x \circ y$ generate a nil-ideal, while the commutators of R of the form $(x \circ y) \circ z$ generate a nilpotent ideal (cf. §3). If R is finitely generated, and $\mathfrak{R}$ is nilpotent then the ideal generated by the commutators $x \circ y$ is also nilpotent (cf. §4).

2. **A lemma on L-nilpotent rings.** We recall that the Lie ring $\mathfrak{R}$ is said to be nilpotent of class γ if we have

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[1] *Central chains of ideals in an associative ring*, Duke Math. J. vol. 9 (1942) pp. 341–355, Theorem 6.5.