CONCERNING AUTOMORPHISMS OF NON-ASSOCIATIVE ALGEBRAS

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In their studies of non-associative algebras A. A. Albert and N. Jacobson have made much use of the relationships which exist between an arbitrary non-associative algebra $\mathfrak{A}$ and its associative transformation algebra $T(\mathfrak{A})$. In this paper we are interested in the automorphism group $\mathfrak{G}$ of $\mathfrak{A}$, and we sharpen the results of Jacobson [**3**, §4][1] and Albert [**2**, §9] in the sense that we prove $\mathfrak{G}$ isomorphic to a *well-defined* subgroup of the automorphism group of each of three associative algebras (§§2, 3).

Incidental to our proofs is the reconstruction (in the sense of equivalence) of an arbitrary non-associative algebra $\mathfrak{A}$ with unity element 1 from $T(\mathfrak{A})$ and from either of the enveloping algebras $E(R(\mathfrak{A}))$, $E(L(\mathfrak{A}))$ of respectively the right or left multiplications of $\mathfrak{A}$. This paper has been expanded in accordance with suggestions of the referee to include a more detailed study of the right ideals used in this reconstruction process (§5).

1. **Preliminaries.** Our notations are chiefly those of Albert as given in [**1**]. We regard a non-associative algebra $\mathfrak{A}$ of order n over a field $\mathfrak{F}$ as consisting of a linear space $\mathfrak{L}$ of order n over $\mathfrak{F}$, a linear space $R(\mathfrak{A})$ of linear transformations R_x on $\mathfrak{L}$ of order $m \leqq n$ over $\mathfrak{F}$, and a linear mapping of $\mathfrak{L}$ on $R(\mathfrak{A})$,

$$x \rightarrow R_x. \tag{1}$$

The elements R_x of $R(\mathfrak{A})$ are called *right multiplications*, and $R(\mathfrak{A})$ the *right multiplication space* of $\mathfrak{A}$. Multiplication in $\mathfrak{A}$ is defined by

$$a \cdot x = aR_x. \tag{2}$$

The linearity of the right multiplications and of (1) insures distributivity in $\mathfrak{A}$ as well as the usual laws of scalar multiplication. We shall use the fact that, in case $\mathfrak{A}$ contains no absolute right divisor of zero (an element x such that $a \cdot x = 0$ for all a in $\mathfrak{A}$), the mapping (1) is nonsingular and the order of $R(\mathfrak{A})$ over $\mathfrak{F}$ is n.

The linear transformations L_x defined by

$$a \rightarrow x \cdot a = aL_x \tag{3}$$

Presented to the Society, August 23, 1946; received by the editors August 2, 1946, and, in revised form, November 13, 1946.

[1] Numbers in brackets refer to the references cited at the end of the paper.