

# SOME PROBLEMS IN CONFORMAL MAPPING

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**1. Introduction.** Attention will be confined to a group of problems centering around so-called schlicht functions—that is, functions regular in a given domain and assuming no value there more than once. The type of problem we consider involves determination of precise bounds for certain quantities depending on the function  $f$ , as  $f$  ranges over the schlicht functions in question. Since, for suitable normalization of the functions at some fixed point of the domain, the resulting family of functions is compact or normal, the extremal schlicht functions always exist and the problem is to characterize them.

Interest was focused on this category of questions by the work of Koebe in the years 1907–1909, who established for the family of functions  $f$  of the form  $f(z) = z + a_2z^2 + a_3z^3 + \dots$ , schlicht and regular in  $|z| < 1$ , a series of properties, among them the theorem of distortion bearing Koebe's name. This theorem asserts the existence of bounds for the absolute value of the derivative  $f'(z)$ , these bounds depending only on  $|z|$ . Further efforts were directed toward finding the precise values of the bounds asserted by Koebe's theorem, but success was not attained until 1916 when Bieberbach, Faber, Pick and others gave a final form to the theorem of distortion. At the same time the precise bound for  $|a_2|$  was given, namely 2, and the now famous conjecture was made that  $|a_n| \leq n$  for every  $n$ . Since 1916 this group of problems has attracted the attention of many, and there is now a considerable literature.

The present state of this sphere of questions will be described briefly in a general sort of way, and a few outstanding problems will be indicated, but no attempt at completeness has been made.

**2. The coefficient problem.** Let  $S$  be the family of functions

$$(2.1) \quad f(z) = z + a_2z^2 + a_3z^3 + \dots$$

which are regular and schlicht in  $|z| < 1$ . The most famous problem concerning these functions is whether  $|a_n| \leq n$  ( $n = 2, 3, \dots$ ), with equality for any  $n$  only in the case when

$$(2.2) \quad f(z) = \frac{z}{(1 - \eta z)^2} = z + 2\eta z^2 + 3\eta z^3 + \dots, \quad |\eta| = 1.$$

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