ON THE STRUCTURE OF INTRINSIC DERIVATIVES

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Introduction. The primary purpose of the present paper is to express the Mth order intrinsic derivative of a higher order absolute tensor such as T^{ab}, T_{ab}, or T^{ab}_{cd} as a contraction of extensors. As a first step, we develop a rule for constructing extensors of the types $E^{\alpha a \cdot \beta b}$, $E_{\alpha a \cdot \beta b}$, $E^{\alpha a : \beta b}_{\gamma c \cdot \delta d}$ from absolute tensors T^{ab}, T_{ab}, T^{ab}_{cd} by repeated differentiation with respect to the curve parameter followed by multiplication by the appropriate coefficients. We then consider the contractions of the various E's with the extended components of connection $L^{e}_{\alpha a}$, $L^{\alpha a}_{e}$ (to be introduced) and prove by induction that these contractions give the Mth order intrinsic derivatives of the original tensors. In this way we establish a highly satisfactory theory of the algebraic structure of the higher order intrinsic derivatives, for the constituent E's and L's obviously possess an invariance of form and being extensors they are such that other extensors, tensors, and invariants can be built from them and other extensors by simple algebraic procedures—addition, multiplication, and contraction.

1. **Notation and preliminaries.** In the present paper we shall employ at most two coordinate systems x and $\bar{x}$ and so far as the quantities that bear indices are concerned, we shall distinguish between them whenever feasible by restricting the choice of indicial letters. Specifically, letters at the first of the alphabet a, b, c, d, e shall serve to denote system x, while r, s, t, u, v, w will be correlated to system $\bar{x}$. Thus x^r is the rth coordinate variable of system $\bar{x}$, while x^a is variable number a of system x. Differentiation with respect to the parameter t of a parameterized arc will be indicated by primes and Greek indices, the latter are enclosed except in certain abridged symbols. To illustrate,

$$x'^{a} = x^{a\prime} = dx^a/dt, \qquad x^{(\alpha)a} = x^{a(\alpha)} = d^{\alpha}x^a/dt^{\alpha},$$
$$X^{\rho r}_{\alpha \rho} = X^{(a)r}_{(\alpha)a} = \partial x^{(\rho)r}/\partial x^{(\alpha)a}, \qquad X^{\alpha a}_{\rho r} = \partial x^{(\alpha)a}/\partial x^{(\rho)r}.$$

Furthermore, we assume that there is given an affine connection L^a_{bc} and let L^a_b, called the two-index connection, represent $L^a_{bc}x'^{c}$.

Summation convention. Repeated lower case Latin indices call for summations 1 to N, while the summations indicated by repeated lower case Greek indices are from zero (unless the contrary is specified) to some terminal value usually M or M+1. Repeated capital Greek in-

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