CURVATURE IN HERMITIAN METRIC

S. BOCHNER

Hermitian metric has the peculiarity of favoring negative curvature over positive curvature. We shall explain this phenomenon by pointing out that in the case of an isometric analytic imbedding the relative curvature is on the whole negative; also, by reduction to a *limiting case* of imbedding we shall explain why an invariant metric in the theory of Fuchsian groups is likely to be hyperbolic; see Hua [6].[1]

However, on the other hand, an Hermitian metric is very rigid, and the possibility of imbedding into a *finitely-dimensional* enveloping space is very remote. The classical conjectures about the possibility of Euclidean imbedding are rendered entirely false, but as a compensation, there are more and better theorems about equivalence and uniqueness.

1. **Hermitian metric.** There are many places in the literature where an introduction to the theory of Hermitian metric can be found. We shall refer to our own summary as given in Bochner [**2**, chap. 2]. We quote from there that in discussing an Hermitian metric in a space V_n of n complex variables $z_1, \cdots, z_n$, the basic variables are the $2n$ conjugate complex quantities

$$z_1, \cdots, z_n; \bar{z}_1, \cdots, \bar{z}_n \tag{1}$$

which we shall also denote indifferently by

$$t_1, t_2, \cdots, t_{2n}. \tag{2}$$

Italic indices run from 1 to $2n$, Greek indices from 1 to n, and starring an index will add to it the value n if it is not greater than n, and subtract n from it if it is not less than $n+1$. All scalars and components of tensors are power series in (1), and they are always *self-adjoint*, meaning that starring all indices in a component of a tensor will change its value into its conjugate complex. Scalars are real-valued.

The fundamental tensor g_{ij} has the properties

$$g_{ij} = g_{ji}, \qquad g_{\alpha\beta} = g_{\alpha^*\beta^*} = 0$$

in addition to the self-adjointness property $g_{\alpha\beta^*} = \bar{g}_{\beta\alpha^*}$; also $g_{\alpha\beta^*}$ is positive-definite. In particular, we have

Received by the editors August 3, 1946.

[1] Numbers in brackets refer to the bibliography at the end of the paper.