ON AREOLAR MONOGENIC FUNCTIONS

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Let $f(z) = u(x, y) + iv(x, y)$, $z = x + iy$, be a complex-valued function defined in the unit circle D: $|z| < 1$. $f(z)$ is said to be *areolar monogenic* in D if and only if $u(x, y)$ and $v(x, y)$ (and hence $f(z)$) have continuous partial derivatives of the second order such that

$$(1) \qquad u_{xy} = -2^{-1}(v_{xx} - v_{yy}), \qquad v_{xy} = 2^{-1}(u_{xx} - u_{yy})$$

hold in D [3].[1] It is known that an areolar monogenic function has partial derivatives of all orders [3].

Whereas (1) is a differential characterization of areolar monogenic functions, it is the integral characterization contained in the following theorem that forms the basis for this note.

THEOREM A [3]. *If $f(z)$ is continuous in D, then a necessary and sufficient condition that $f(z)$ be areolar monogenic in D is that there exist a function $g(z)$, analytic in D, such that*

$$(2) \qquad g(z) = \frac{1}{\pi r^2 i} \int_{C(z;r)} f(\zeta) d\zeta, \qquad \zeta = \xi + i\eta,$$

holds for all circles $C(z; r)$, with center z and radius r, in D.

It should be noted that a symbol once introduced holds its meaning throughout the paper.

If $f(z)$ is continuous in D, then the right-hand member of (2) is a function of z and r, defined for z in the circle D_r: $|z| < 1 - r$, and for all r such that $0 < r < 1$. Now if the definition

$$(3) \qquad G_r(z) \equiv \frac{1}{\pi r^2 i} \int_{C(z;r)} f(\zeta) d\zeta, \qquad 0 < r < 1, \ |z| < 1 - r,$$

is made, then the following is an extension of Theorem A.

THEOREM 1. *If $f(z)$ is continuous in D, then a necessary and sufficient condition that $f(z)$ be areolar monogenic in D is that $G_r(z)$ be analytic in D_r, for all r, $0 < r < 1$.*

Necessity. This is precisely the necessity part of Theorem A.

Sufficiency. Suppose that in addition $f(z)$ has continuous partial

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[1] Numbers in brackers refer to the bibliography at the end of the paper.