84. Edward Kasner and John DeCicco: *Rational harmonic curves.*

A rational harmonic curve is defined by setting the real part of a rational function of $x+iy$, of degree r, equal to zero. Any such curve is given by $P(x, y)=0$, where P is a polynomial of (x, y) of degree $2r-k$, where $0 \leqq k \leqq r$. The curve has k real asymptotes, all of which pass through a fixed point and which make equal angles with each other. The angle between consecutive asymptotes is π/k. The remaining asymptotes are minimal and $2(r-k)$ in number. The theorem generalizes to rational harmonic curves a theorem of Briot and Bouquet concerning the asymptotes of polynomial harmonic curves, which are defined by setting the real part of any rational integral function of a complex variable equal to zero. Other properties are given by means of the foci of systems of confocal curves. Additional results are found about satellites. (Received October 4, 1946.)

85. C. E. Springer: *Union torsion of a curve on a surface.*

The geodesic torsion at a point of a curve on a surface is the torsion of the geodesic which is tangent to the curve at the point. In this paper the union torsion at a point of a curve C on a surface is defined as the torsion of the union curve in the direction of the curve C, the union curve being defined relative to a given rectilinear congruence. The union torsion is given by a formula which reduces to the expression for geodesic torsion in case the congruence is normal to the surface. It is shown that a union curve is a plane curve if, and only if, it is tangent to a curve of intersection of a developable of the congruence with the surface. (Received October 3, 1946.)

86. A. E. Taylor: *A geometric theorem and its application to biorthogonal systems.*

Let S be a bounded and closed point set in E_n (Euclidean space of n dimensions). Let O be a point such that O and S together are not contained in any subspace of $n-1$ dimensions (such a subspace is hereafter called a plane). Then there exist n linearly independent vectors $x_1, \cdots, x_n$ emanating from O, with terminal points $P_1, \cdots, P_n$ in S, and n planes $p_1, \cdots, p_n$ satisfying the following conditions: (a) p_i contains P_i; (b) p_i is parallel to the plane determined by $x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n$; (c) S and O both lie in the same one of the two closed half-spaces into which p_i divides E_n. This theorem is proved, and then applied to demonstrate the following: If Y_n is an n-dimensional subspace of a normed linear space X, there exist n elements $x_1, \cdots, x_n$ of unit norm in Y_n and n linear functionals $f_1, \cdots, f_n$ of unit norm defined on X, such that $f_i(x_j)=\delta_{ij}$. (Received October 25, 1946.)

Statistics and Probability

87. G. E. Forsythe: *On Nörlund summability of random variables to zero.*

The present paper is an incomplete extension to regular Nörlund summability methods of some previous results (G. E. Forsythe, Duke Math. J. vol. 10 (1943) pp. 397–428, §5) on Cesàro summability in probability of random variables to zero. Corresponding to a sequence p_0 $(=1)$, $p_1, p_2, \cdots$ of non-negative constants, the Nörlund method N_p is defined by the triangular Toeplitz matrix $\|a_{nk}\|$, where $a_{nk} = p_{n-k}(\sum_{k=0}^{n} p_k)^{-1}$. It is conjectured that if $N_p \subset N_q$ with respect to the summability of