A GENERALIZATION OF A THEOREM OF LEROY AND LINDELÖF

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1. **Introduction.** Consider a Taylor series $f(z)=\sum_{n=0}^{\infty}a_nz^n$ with radius of convergence unity. Let the coefficients a_n be the values taken on by a regular function $a(z)$ for $z=0, 1, \cdots$.

The object of this paper is to study the Taylor series under the assumption that $a(z)$ is regular in certain domains.[1] The results obtained are of the nature of domains in which the function defined by $\sum_{n=0}^{\infty}a_nz^n$ is regular and of domains which contain the singularities of the function defined by the series. In terms of $a(z)$ fairly general sufficient conditions are given such that the circle of convergence is not a cut for the function.

The results may be regarded as a generalization of a theorem due to LeRoy and Lindelöf.[2]

THEOREM (LEROY AND LINDELÖF). *Suppose* (a) $a(x+iy)$ *is regular in the semiplane* $x\geqq\alpha$, (b) *there is a* $\theta<\pi$ *such that for every arbitrary small positive* ϵ *and for sufficiently large* ρ

$$|a(\alpha+\rho\exp(i\psi))| < \exp[\rho(\theta+\epsilon)], \qquad -\pi/2\leqq\psi\leqq\pi/2.$$

Then

$$f(z)=\sum_{n=0}^{\infty}a(n)z^n, \qquad z=r\exp(i\phi)$$

is regular in the angle

$$\theta<\phi<2\pi-\theta.$$

The generalization of this theorem that we prove consists, under suitable restrictions, in replacing the semiplane $x\geqq\alpha$ by an angular opening including the axis of positive reals in its interior.

The singularities of the function $f(z)$ studied in this paper are those of a "principal branch" obtained by immediate continuation of the series.

Consider an angular opening with vertex on the positive real axis which includes the axis of reals in its interior. Suppose $a(z)$ has no singularities in this angular opening with the possible exception of

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[1] By the term domain we mean an open connected set.

[2] See Dienes [1]. Numbers in brackers refer to the bibliogiaphy at the end of the paper.