preceding abstract (with $\alpha\beta$ replaced by α/β). B_3. Multiplication is commutative. C. The dyads of T, including ω/ξ, form a commutative group under the undefined compostion "addition." Under C postulates are first stated (C_1) in terms of addition of dyads with the same posterior elements (denominators); it is then assumed (C_2) that α/β, γ/δ in T imply the existence of ξ such that $\gamma/\delta=\xi/\beta$. Definition: $\alpha/\beta+\gamma/\delta=\lambda/\mu$ means: There exist ξ, η such that $\gamma/\delta=\xi/\beta$, $\eta/\beta=\lambda/\mu$, where $\alpha/\beta+\xi/\beta=\eta/\beta$. D. Multiplication over addition to the right (left) is distributive. The independence of C_2 is discussed. Application is made to the author's *Foundations of Grassmann's extensive algebra* (Amer. J. Math. vol. 35 (1913) pp. 39–49). (Received September 27, 1946.)

354. J. D. Swift: *Periodic functions over a finite field.*

A function of period a over the Galois field, $GF(p^n)$, is defined as a function over the elements of the field to the elements of the field such that $f(x+na)=f(x)$, where n is an integer. Multiply periodic functions are defined in an analogous manner. A $GF(p^n)$ admits functions with a number of independent periods not greater than $n-1$. The basic function of period a is: $f(a;x)=x^p-a^{p-1}x$. The basic function of periods $a_1, \cdots, a_n$ may be defined recursively from the above as: $f(f(a_1, \cdots, a_{n-1}; a_n)$; $f(a_1, \cdots, a_{n-1}; x))$. These basic functions are odd and additive, and $f(a_1, \cdots, a_n; cx)$ is periodic with periods $a_1/c, \cdots, a_n/c$. The principal result is: *Any periodic function of periods* $a_1, \cdots, a_k$ *over the* $GF(p^n)$, $k\leqq n-1$, *may be expressed by*: $g(x)=\sum_1^l \alpha_i f^i(a_1, \cdots, ak; x)+\alpha_0)$, where $l=p^{n-k}-1$, *and the* α_i *are a set of elements of the field.* (Received September 16, 1946.)

355. P. M. Whitman: *Finite groups with a cyclic group as lattice-homomorph.*

It is shown that if G and H are groups, $L(G)$ and $L(H)$ their lattices of subgroups, $L(G)$ is finite, $L(H)$ is a lattice-homomorphic image of $L(G)$, and H is cyclic, then G contains a cyclic subgroup which is mapped onto H by the homomorphism. (Received September 26, 1946.)

Analysis

356. Warren Ambrose: *Direct sum theorem for Haar measures.*

A variation of a theorem of A. Weil (*L'intégration dans les groupes topologiques*, Paris, 1940, pp. 42–45) is proved. (Received September 19, 1946.)

357. R. F. Arens: *Location of spectra in Banach $*$-algebras.*

A Banach $*$-algebra A is a Banach space with a continuous multiplication and a $*$-operation satisfying $(\lambda f+\mu g)^*=\bar{\lambda}f^*+\bar{\mu}g^*$, $f^{**}=f$, $(fg)^*=g^*f^*$, and $k|f|\,|f^*|\leqq|ff^*|$, $k>0$. One can renorm A such that $\|fg\|\leqq\|f\|\,\|g\|$, and there will exist $k'>0$ such that $k'\|f\|\,\|f^*\|\leqq\|ff^*\|$. It is proved that, if $f=u+iv$, $u^*=u$, $v^*=v$, $uv=vu$, then $|x\cos\theta+y\sin\theta|\leqq\|u\cos\theta+v\sin\theta\|$ for any complex number $x+iy$ in the spectrum of f, and $0\leqq\theta\leqq2\pi$. Thus if $f=f^*$, the spectrum is real. The case $k'=1$ studied by I. Gelfand and M. Neumark, Rec. Math. (Mat. Sbornik) N. S. vol. 12 (1943) pp. 197–213, is a special case. (Received August 8, 1946.)

358. Lipman Bers: *A property of bounded analytic functions.*

Let $f(z)$ be bounded and analytic for $|z|<1$. If $\{z_n\}$ is a sequence of points such